Input-Output Linkages and Productivity Measurement

Hang Hu*

University of Melbourne

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Abstract

This paper studies the role of input-output (IO) linkages in sectoral and aggregate productivity measurement for 39 countries during 1995-2007. The main finding is that without taking IO linkage into account, development accounting (DA) method biasedly measure the sectoral TFP and sectoral labor productivity for all sample countries. For aggregate TFP, the quantitative analysis result suggests that compared with IO based measurement, DA method overestimates annual aggregate TFP growth rate by about 15 percent for the US and underestimates that by about 9 percent for China. For the 39 sample countries on average, DA underestimates the annual aggregate TFP growth rate by about a half percent. Moreover, the counterfactual study implies that previous studies are either insufficient to taking IO linkage into account, or totally ignore IO linkage which result in biased productivity measurement and implication.

Keywords: Input-Output Linkage, Productivity Measurement, TFP, Labour Productivity.

JEL classification: E01, O1, O4.

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1. Introduction

Recent productivity slowdown of US and many OECD countries have caused a debate about whether the slowdown is due to measurement problem. [1] Though the underestimate of IT-related product and quality improvement may contribute to the measurement error, the estimate structure or method itself could generate measurement error as well. For simplicity, many economists measure productivity without taking into account the abundant input-output (IO) linkage. They directly use value added as the net final product; and they simply apply development accounting method to measure productivity. Both Academics and industry analysts take the ignorance of IO linkage as granted, without asking whether IO linkage affects productivity measurement. This paper seeks to answer this question.

The question of how does IO linkage affects productivity measurement is important for at least two reasons. First, productivity plays a Central role in income growth and income difference in the long run. According to Jones (2015), the total factor productivity (TFP) growth accounts for about 80 percent of income (GDP per hour) growth of US since 1948. In addition, Jones (2015) estimates that for middle and low income countries, such as China and Kenya above 80 percent of the difference in GDP per worker relative to US is due to TFP difference; for high income country such as France and Germany, TFP contributes to about 50 percent of difference. Given the well-known importance of productivity, we need to measure it as unbiasedly as possible. Second, sectors are not independent in the real world. Their production are connected by IO networks. Actually they do not produce value added directly. They produce gross output, and then their gross output are supplied to other sectors as intermediate inputs, or supplied to consumers as final goods. If we ignore the role of intermediate input supply and demand, the sectoral importance relative to the whole economy may be misunderstood. Moerover this sectoral importance determines the sectoral weight, and the weight matters for sectoral productivity measurement and therefore also matters for aggregate productivity measurement.

In this paper I study the effect of IO linkage on sectoral and aggregate productivity measurement. In order to identify the IO linkage effect, I compare the productivity mea-

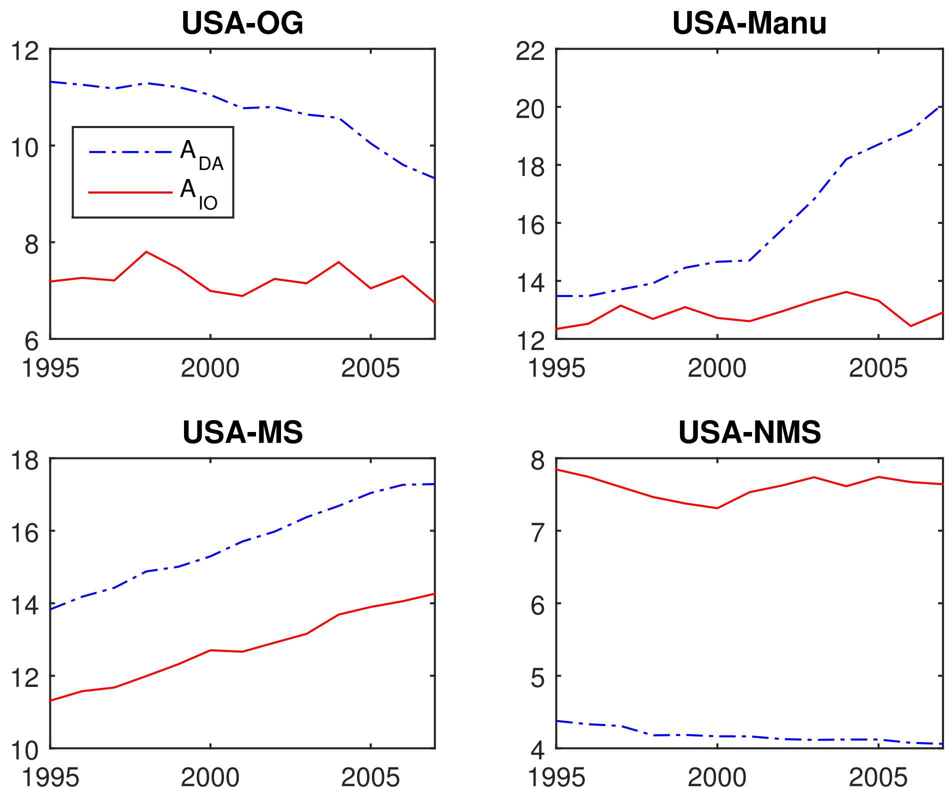
[1] Syverson (2017) provides a good review on this debate.

surement under two cases. In the first case I take IO linkage into account and then measure value added based sectoral productivity and aggregate productivity. In the second case I imagine that an econometrician measures the value added based sectoral and aggregate productivity directly by using development accounting method. I compare these two measurements. I check whether they are different. If they are different, I further study where the difference come from. Finally I compare my IO based productivity measurement with previous two strands of literature. For the literature which also takes into account IO linkage to measure productivity, I study whether my measurement is consistent with them. For the literature which measures productivity without IO linkage, I study how the IO linkage based measurement may change their findings.

I find that IO linkage has significant effect on sectoral and aggregate productivity measurement. First, the IO linkage based aggregate productivity measurement are close to the development accounting based aggregate productivity measurement without IO linkage. Because the aggregate factor shares are still close even if ignoring IO linkage. Given the same aggregate output and input, the aggregate productivity measurements are close. However, sectoral productivity measurements could be very different between these two cases. Because the value added (VA) based sectoral weights are endogenous on IO linkage. If we ignore IO linkage, we misinterpret the true sectoral weight. Given the close measurement of aggregate productivity, biased sectoral weight implies biased sectoral productivity measurement. Figure 1 shows the different sectoral TFP measurement of US under IO based measurement (from the model result) and DA based measurement. The four sectors are: other good (OG); Manufacturing (Manu); market service (MS) and non-market service (NMS). The sector classification is presented in table 7 in appendix B. The result suggests significantly biased productivity measurement without considering IO linkage. Moreover, though the aggregate productivity measurement are close between the two cases, the aggregate productivity growth measurement could be biased remarkably if ignoring IO linkage. Biased sectoral productivity growth measurement causes biased measurement of within sectoral TFP growth and structural change, which finally causes biased measurement of aggregate productivity growth.

Following Jones (2011), I develop a multi-sector general equilibrium model with exogenous IO linkage and sectoral TFP. The aggregate real output and aggregate TFP could be solved analytically. Then I construct a corresponding case which ignores IO linkage

Figure 1: The sectoral TFP measurement of US with and without IO linkage



Note: This figure shows the sectoral TFP measurement under two case: development accounting (DA) based measurement and input-output (IO) based measurement.

and directly taking value added into account. I compare the aggregate TFP between these two cases: gross output based TFP and value added based TFP. The sectoral weight in IO economy is Domar weight (Domar, 1961) which is defined as a ratio of sectoral gross output value to aggregate GDP. Domar weight is endogenous on IO linkage. The more intermediate input suppliable, the larger Domar weight in this sector. The corresponding sectoral weight in VA economy is sectoral value added share. I find that these two aggregate TFPs are observationally equivalent as long as sectoral value added share is endogenous on the corresponding Domar weight. It implies that VA economy could exactly replicates the IO economy in aggregate level if allow value added share specifically endogenous on IO linkage.

Based on the aggregate TFP equivalence result, I construct the model implied and

VA based sectoral TFP measurement. The aggregate TFP equivalence result implies a one-to-one mapping from gross output based sectoral TFP to VA based sectoral TFP. I call the measurement of VA based sectoral TFP which is mapped from the corresponding gross output based sectoral TFP as IO based sectoral TFP measurement (also called model implied measurement). I call the the measurement of VA based sectoral TFP which is directly calculated from development accounting method as DA based sectoral TFP measurement (also called measurement from econometrician). I compare IO based sectoral TFP with DA based sectoral TFP. I find that both price difference and IO structure contribute to the measurement difference at sectoral level.

My quantitative analysis depends mainly on two strands of database. Data of sectoral nominal values and factor shares are from World Input-Output Database (WIOD) which is developed by Timmer, Dietzenbacher, Los, Stehrer and de Vries (2015). Data of PPP price deflators are from Sector and Industry Relative Prices (SIRP) which is developed by Inklaar and Timmer (2014). By using these two databases, I can calculate the real value of sectoral output and input. Then I can construct the sectoral productivity panel data. The final productivity data contains 4 sectors over 1995-2007, for 39 countries which includes 20 developed countries and 19 developing countries.

The result of quantitative analysis is consistent with the model prediction. First, the DA based sectoral TFP measurements could be substantially divergent to the IO based sectoral TFP measurement. This divergence happens to almost every sector and to every sample country. Figure 1 shows an example for US. Second, the DA based aggregate TFP growth decomposition measurement could be biased as well, compare with that of IO based decomposition. Given the sectoral TFP growth measurement bias, there is no surprising to see measurement bias in aggregate TFP growth decomposition. At the same time, it implies aggregate TFP growth measurement bias. Third, the result and conlcusion of many previous literatures which study cross-country sectoral or aggregate productivity difference without IO linkage would likely to be reversed, if those productivities are measured by taking into account the IO linkage. The result of counterfactual study suggests that IO linkage has crucial implication on productivity comparison research.

1.1 Literature

This paper connects to four strands of literature of productivity. The first two literatures take IO linkage into account. Another two literatures totally ignore IO linkage.

First, my paper is most closely connected to literature which discusses the IO effect on sectoral productivity measurement. As far as I know, Duarte and Restuccia (2016) is the first paper which mentions the IO effect on sectoral productivity measurement. They develop a three-sector model with IO linkage which builds on Ngai and Samaniego (2009). They figure out the sectoral nominal value added as the difference between sectoral gross output value and sectoral intermediate input value. Different to my model result, they directly define the sectoral value added based productivity from the equation of nominal value added. However, this definition is problematic since they cannot really separate out the true sectoral TFP from the corresponding sectoral value added price. They can correctly derive the product of productvity and price; but they cannot isolate which term belongs to productivity and which term belongs to price. My model result can clearly derive the VA based sectoral productivity which is a function on the corresponding gross output based sectoral productivity and IO linkage structural parameters. Actually their definition of sectoral VA based productivity is incorrect. Because Ngai and Samaniego (2009) also defines the VA based sectoral productivity, and they define in a different way to Duarte and Restuccia (2016). While Ngai and Samaniego (2009) chooses to define the VA based sectoral productivity as the product of IO parameter adjusted gross output based productivity and an additional IO linkage strucutre term; Duarte and Restuccia (2016) simply call the former term as VA based sectoral TFP. The productivity definition of Ngai and Samaniego (2009) is a special case of my model result. Therefore, even though Duarte and Restuccia (2016) mentions the effect of IO linkage on sectoral productivity measurement, their identification strategy is insufficient and their result is inconsistent with my result.

Second, my paper connects to literature which comprehensively compare IO economy and VA economy. In theory I prove the observationally equivalent aggregate TFP between IO economy and the corresponding VA economy. But this equivalence result depends on a specifically defined value added share. This equivalence result is consistent with Leal (2015) which shows the equivalent degree of influence between IO economy and VA economy. But Leal (2015) has no discussion about the implication of this equivalence result. In particular he has no discussion about how does the equivalence result implies sectoral productivity measurement. The equivalence result could be taken as a special case of Domar (1961) and Balk (2009). These two papers argue that aggregate TFP are divergent between IO economy and VA economy if production function is in a general form or if value added share is exogenously defined.

Third, my paper follows literature which decomposes aggregate productivity growth. They decompose the aggregate productivity growth into two terms: within sectoral productivity growth and cross sectoral structural change (McMillan et al. 2014, 2017). They use this decomposition to show that the heterogeneous structural change across countries plays a role in heterogeneous aggregate productivity growth across countries. In my paper, I have no discussion about this relationship. Because my paper focus on the measurement of aggregate productivity growth. In my paper, the decomposition is used to show that the biased sectoral TFP growth measurement would cause biased measurement of both decomposition terms, which finally cause biased aggregate productivity growth measurement. In section 5, I show that their result of decomposition are biased if measuring productivity without IO linkage.

Last, my paper points out the weakness of previous literature which directly uses development accounting method to measure productivity, and apply the measurement to explain productivity difference across countries. Bernard and Jones (1996), Young (1995) and Herrendorf and Valentinyi (2012) are good examples of this literature, among other researches. They all measure productivity without accounting of IO linkage. So their productivity measurements are subject to the measurement bias. More importantly their conclusions which build on the biased productivity measurement, are likely to be misunderstood. In section 5 I conduct counterfatual study by replacing DA based productivity measurement with IO based productivity measurement. The result suggests that their conclusion could be reversed, once IO linkage is considered.

The paper is outlined as follows. In section 2 I introduce the model and show the model result of aggregate productivity equivalence; model implied sectoral productivity measurement; and the difference between model implied sectoral productivity measurement and the corresponding development accounting based measurement. I introduce the data and also discuss how I deal with the data in section 3. Then in section 4 I apply the model result to the data. I show the quantitative analysis of sectoral TFP measurement bias, the source of bias, and the aggregate TFP decomposition bias. In section 5 I discuss the quantitative implication of IO based productivity measurement on the result of past literatures. I conclude in section 6.

2. Model

I present the model in this section. The model with IO linkage is presented in section 2.1. Given IO linkage, production output is gross output and intermediate inputs are explicitly taken into account in the production function. The corresponding case of VA economy is illustrated in section 2.2. Then in section 2.3, I show the aggregate TFP equivalence result between these two cases. While aggregate TFPs are observationally equivalent, the sectoral VA based TFP measurement may significnatly different between the model implied result (IO based) and the DA implied result. In section 2.4 I discuss how IO linkage affects sectoral TFP measurement difference.

2.1 Model of IO Economy

The model follows Jones (2011), without international trade and distortion. Assume there are n sectors in an economy. The sectoral gross output is produced in a constant return to scale Cobb-Douglas function form.

$$Q_i = A_{GOi} K_i^{(1-\sigma_i)\alpha_i} L_i^{(1-\sigma_i)(1-\alpha_i)} \prod_{j=1}^{n} X_{ij}^{\sigma_{ij}} \tag{1}$$

Here $\sigma_i = \sum_{j=1}^{n} \sigma_{ij}$; K_i and L_i stand for capital and labour demand of sector i respectively; X_{ij} represents the intermediate input produced from sector j, and used by sector i; A_{GOi} is gross output based total factor productivity for sector i. The gross output is either consumed as final goods or supplied to other sectors as intermediate inputs.

$$C_j + \sum_{i=1}^{n} X_{ij} = Q_j \tag{2}$$

The aggregate final goods (real GDP) is assumed to be Cobb-Douglas composite of each individual sectoral consumption, such that

$$Y = \prod_{i=1}^{n} C_i^{\lambda_i} \tag{3}$$

Here λ_i is consumer preference of consumption from sector i and $\sum_{i=1}^{n} \lambda_i = 1$.

In addition, assume a representative consumer's aggregate utility function is Cobb-Douglas function of sectoral consumption, as specified in equation (3). Therefore, the model implies a unity intertemporal elasticity of substitution and also a unity elasticity of substitution between sectoral consumtions. In a static competitive equilibrium, consumer maximize utility by choosing optimal consumption from each sector. The representative consumer's problem in sector i is

$$\max_{C_i} \prod_{i=1}^{n} C_i^{\lambda_i} - \sum_{i=1}^{n} P_i C_i$$

The corresponding first order condition of consumer's problem is

$$\lambda_i \frac{Y}{C_i} = P_i \tag{4}$$

No surprisingly, the representative consumer spends λ_i of aggregate income in consumption from sector i. In IO economy, sectoral consumptions are observable. The aggregate value added equals the sum of sectoral consumption.

Moreover for each sector, assume there is a representative firm to maximize sectoral profit by choosing optimal capital, labour and intermediate input. The representative firm's problem in sector i is

$$\max_{K_i, L_i, X_{ij}} P_i A_{GOi} K_i^{(1-\sigma_i)\alpha_i} L_i^{(1-\sigma_i)(1-\alpha_i)} \prod_{j=1}^{n} X_{ij}^{\sigma_{ij}} - rK_i - wL_i - \sum_{j=1}^{n} P_j X_{ij}$$

The corresponding first order conditions of choosing capital, labour and intermediate inputs respectively are

$$(1-\sigma_i)\alpha_i \frac{P_i Q_i}{K_i} = r \tag{5}$$

$$(1-\sigma_i)(1-\alpha_i)\frac{P_iQ_i}{L_i} = w \tag{6}$$

$$\sigma_{ij}\frac{P_iQ_i}{X_{ij}} = P_j \tag{7}$$

In equilibrium, assume that capital market and labour market are clear. Assume capital and labour are supplied inelastically. Then the market clear conditions are given by

$$\sum_{i=1}^{n} K_i = K \tag{8}$$

$$\sum_{i=1}^{n} L_i = L \tag{9}$$

There are 9 equations to solve 9 variables in equilibrium: $Y, Q_i, K_i, L_i, X_{ij}, C_i, P_j, w, r$.

After solving the model (all proofs of this and other propositions are shown in appendix A), the following proposition summarises the key result:

Proposition 1 (Equilibrium Solution of IO Economy): *Under competitive equilibrium, aggregate value added and aggregate TFP are solved as*

$$Y = TFP_{GO}K^{\hat{\alpha}_{GO}}L^{1-\hat{\alpha}_{GO}} \tag{10}$$

$$logTFP_{GO} = \gamma' \, logA_{GO} + \lambda' W_c + \gamma' W_{GOq} \tag{11}$$

where the relevant terms and parameters are defined as following:
Domar weight: $\gamma = (I-B)^{-1}\lambda$; $\gamma_i = \frac{P_iQ_i}{Y} = \sum_{j=1}^{n} l_{ij}\lambda_j$; *Aggregate Capital share:* $\hat{\alpha}_{GO} = \gamma'[(1-\sigma)\circ\alpha]$; *Allocation efficiency terms:* $W_c = log\lambda - log\gamma$; $W_{GOqi} = (1-\sigma_i)\alpha_i log\theta_{GOKi} + (1-\sigma_i)(1-\alpha_i)log\theta_{GOLi} + \sum_{j=1}^{n}\sigma_{ij}log\theta_{ij}$; *Allocation parameters:* $\theta_{GOKi} = \frac{(1-\sigma_i)\alpha_i\gamma_i}{\sum_{i=1}^{n}(1-\sigma_i)\alpha_i\gamma_i}$; $\theta_{GOLi} = \frac{(1-\sigma_i)(1-\alpha_i)\gamma_i}{\sum_{i=1}^{n}(1-\sigma_i)(1-\alpha_i)\gamma_i}$; $\theta_{ij} = \sigma_{ij}\frac{\gamma_i}{\gamma_j}$.

Here TFP_{GO} is gross output based aggregate TFP. The aggregate TFP is a weighted sum of sectoral TFP and importantly, the sectoral weight is Domar weight. This result is consistent with literature which argues that aggregate TFP is a Domar weighted sum of sectoral TFP in the IO economy (Domar 1961; Hulten 1978; Carvalho and Gabaix 2013; Acemoglu et al. 2017). Since the sum of Domar weights is normally larger than 1, the aggregate TFP is greater than the average value of sectoral TFPs. The intuition is summarised in Domar (1961) and Hulten (1978). Given sectoral TFP increases, it increase both value added and intermediate inputs. Those additional intermediate inputs are supplied to other sectors, which further increases the value added and intermediate inputs and the process continues. In effect, the weight of individual sector depends on the ability of supplying final goods and intermediate inputs. The sectoral weight is the ratio of sectoral gross output to GDP rather than sectoral value added to GDP. Therefore this result emphasizes that the importance of a sector in the input-output economy, the importance depends on its supply ability, in particular depends on the role played as a intermediate input supplier to other sectors.

If the role of intermediate input supply of a sector to other sectors changes, the domar weight varies. This is the gross output based structural transformation. The equation (11) also implies that the aggregate TFP contains the Domar weighted sum of input allocation efficiency. Since there is no misallocation (Hsieh and Klenow 2009), the efficiency term W_{GOq} describe the value under optimal allocation. The efficiency value depends on the employment share, capital allocation and intermediate inputs allocation. These values response to the Domar weight change, which is called the input structural transformation. Therefore, in a times series sense, Domar weight affects the aggregate TFP change directly and also affect it indirectly through the effect on input structural transformation. The Domar weight plays the role as a intermediate agent between the intermediate input supply importance and the input allocation efficiency. In the later section, I show that the Domar weight also likely to affect the measured value of sectoral value added based TFP. Hence conclude that the Domar weight plays a crucial role in connecting the IO linkage and aggregate TFP.

2.2 Corresponding Case of VA Economy

Conventionally people ignore the IO linkage to avoid double counting output. If we do not consider IO linkage, the sectoral value added is assumed to be a constant return to scale Cobb-Douglas production function of primary inputs capital and labour:

$$Y_i = A_i K_i^{\beta_i} L_i^{(1-\beta_i)} \tag{12}$$

Correspondingly here A_i is value added based total factor productivity for sector i. The aggregate value added (real GDP) is assumed to be a Cobb-Douglas composite of each individual sectoral value added.

$$Y = \prod_{i=1}^{n} Y_i^{\eta_i} \tag{13}$$

Here η_i is the value added share in sector i and $\sum_{i=1}^{n} \eta_i = 1$. In the value added economy the sectoral value added is observable. The aggregate value added now is sum of sectoral value added in the production side. The first order conditions of consumer's problem and firm's problem now becomes to

$$\eta_i \frac{Y}{Y_i} = P_{Yi} \tag{14}$$

$$\beta_i \frac{P_{Yi} Y_i}{K_i} = r \tag{15}$$

$$(1-\beta_i) \frac{P_{Yi} Y_i}{L_i} = w \tag{16}$$

After solving the corresponding VA economy, the following proposition summarises the key result:

Proposition 2 (Equilibrium Solution of VA Economy): *Under the competitive equilibrium, the aggregate value added and aggregate TFP are solved as*

$$Y = TFP_{VA} K^{\hat{\alpha}_{VA}} L^{1-\hat{\alpha}_{VA}} \tag{17}$$

$$log TFP_{VA} = \eta' \, log A + \eta' W_{VAq} \tag{18}$$

where the relevant terms and parameters are defined as following: Aggregate Capital share: $\hat{\alpha}_{VA} = \eta' \beta$ *Allocation efficiency term:* $W_{VAqi} = \beta_i log \theta_{VAKi} + (1-\beta_i) log \theta_{VALi}$ *Allocation parameters:* $\theta_{VAKi} = \frac{\beta_i \eta_i}{\sum_{i=1}^{n} \beta_i \eta_i}$*;* $\theta_{VALi} = \frac{(1-\beta_i)\eta_i}{\sum_{i=1}^{n}(1-\beta_i)\eta_i}$

Compared to the aggregate TFP equation (11) in the input-output economy, the aggregate TFP in the value added economy is a value added share weighted sum of value added based sectoral TFP and the primary input allocation efficiency.

2.3 Aggregate TFP Equivalence

Normally the capital share in value added should be equal in these two cases. It implies that $\alpha_i = \beta_i$. Given $P_{Yi} Y_i = (1-\sigma_i) P_i Q_i$, the optimal sectoral capital stock equals under these two cases according to the equation (5) and equation (15). It also implies that sectoral capital allocation are equivalent between these two cases. These observational relations together with proporsition 1 and proporsition 2 implies that the following proposition holds:

Proposition 3 (Aggregate TFP equivalence): *If $\alpha_i = \beta_i$ and $P_{Yi}Y_i = (1-\sigma_i)P_iQ_i$, we have*

$$\eta_i = (1-\sigma_i)\gamma_i \tag{19}$$

$$TFP_{VA} = TFP_{GO} \tag{20}$$

If the sectoral value added in the value added economy is the difference between the gross output and total intermediate inputs in the input-output economy; and suppose the sectoral capital share equals under these two economies, the sectoral value added share is endogenously defined as product of sectoral primary input share and the corresponding Domar weight. Then the aggregate TFP in the value added economy exactly equals the aggregate TFP in the input-output economy.

This result is similar to Leal (2015) which shows that under Cobb-Douglas production function and competitive equilibrium, the IO economy is equivalent to VA economy. But the equivalence result in Leal (2015) is emphasized in the degree of influence, not mentioned to productivity. Most importantly, Leal (2015) has nothing to say about the implication of equivalence on productivity measurement. Notice that the aggregate TFP based on the IO economy is not always guaranteed to equal the aggregate TFP based on the VA economy. Under a general production function Balk (2009) or under a Cobb-Douglas production function but with exogenously defined valued added share (Domar 1961), the aggregate TFP under these two economies are divergent. Therefore this observational equivalence at aggregate level is a special case when the production function is CRS Cobb-Douglas; aggregate capital stock and labour supply are perfectly inelastic; production market is perfectly competitive and no distortion in input and output market. The equivalence result may violate if one of this assumption not holds.

To understand the intuition behind this equivalence result, I return to the equations of aggregate GDP under these two cases. The equations for aggregate real GDP under these two cases are illustrated in equation (10) and equation (17). The difference of these two aggregate output equations are aggregate capital share and aggregate TFP. If the aggregate capital share equals, the aggregate TFP has to equal. The aggregate capital share is weighted sum of sectoral capital share. The sectoral weight in the IO economy is Domar weight; and the sectoral weight in VA economy is value added share. The sectoral weights are diffrent between these two economies; and the sectoral total capital shares are different between these two economies as well. Nevertheless, the product of sectoral weight and sectoral capital share are equivalent between IO economy and VA economy. That is,

$$\eta_i \delta_{VAKi} = \eta_i \alpha_i = \gamma_i (1 - \sigma_i) \alpha_i = \gamma_i \delta_{GOKi}$$

The equivalent weighted sectoral capital shares at every sector implies the sum of weighted sectoral capital shares must be equal between the two economies. Therefore the aggregate capital shares are equal, which implies equivalent aggregate TFP between the input-output economy and the value added economy.

2.4 Sectoral TFP Measurement Bias

Though the aggregate TFP is equivalent between IO economy and VA economy, the value added based sectoral TFP measurement could be biased without considering the input-output linkage. Suppose an econometrician measure the sectoral TFP directly by using the development accounting method such as equation (12). On the other hand, I measure the gross ouput based sectoral TFP first based on equation (1), and then calculate the value added based sectoral TFP based on the model implied equivalence result. I show the difference between these two measurement results in this section. More importantly I show where the difference comes from and which implication I can find from the measurement difference.

The aggregate TFP equivalence from Proporsition 3 implies that the relationship between sectoral value added based TFP and gross output based TFP is

$$\eta' \log A_{\text{VA}} = \lambda' (I - B')^{-1} \log A_{\text{GO}} + \lambda' (I - B')^{-1} W_{\text{GO}q} + \lambda' W_c - \eta' W_{\text{VA}q} \tag{21}$$

Basically equation (21) repeats the aggregate TFP equivalence result between IO and the VA economy. Substitute the definition of value added share from equation (19) to equation (21), I have

$$(1-\sigma_i)\log A_{\text{VA}i} = \log A_{\text{GO}i} + ((I-B')W_c)_i + \sum_{j=1}^{n}\sigma_{ij}\log\theta_{ij} \tag{22}$$

Given the input-output structure, equation (22) shows a sector-to-sector mapping from gross output based TFP to value added based TFP. If not considering the IO linkage, assume the sectoral TFP is measured by development accounting method. Normally an econometrician assumes a production function such as the Cobb-Douglas production functuon in equation (12). Combine equation (12) and equation (1), the relationship between value added based sectoral TFP measured from development accounting method and gross output based sectoral TFP is

$$(1-\sigma_i)\log A_{\text{DA}i} = \log A_{\text{GO}i} + (1-\sigma_i)\log(1-\sigma_i) + \sum_{j=1}^{n}\sigma_{ij}(\log\sigma_{ij} + \log P_i - \log P_j) + (1-\sigma_i)(\log P_i - \log P_{Yi}) \tag{23}$$

To calculate the value added based sectoral TFP by development accounting method, I do not need to consider the gross output based sectoral TFP. For comparison convenience, I build up the relationship between the development accounting method implied value added based TFP and the gross output based TFP in equation (23). Here I use $A_{\text{DA}i}$ to represent the value added based sectoral TFP which is derived from the development accounting method. For comparison convenience I label the model implied value added based sectoral TFP as $A_{\text{IO}i}$, which measured from equation (22). Based on these two measurement equation (22) and equation (23), the difference between these two measured sectoral TFP in logarithm form is

$$\log A_{\text{DA}i} - \log A_{\text{IO}i} = \log P_i - \log P_{Yi} + \frac{1}{1-\sigma_i}\left(\sum_{j=1}^{n}\sigma_{ij}(\log P_i - \log P_j)\right) + \log(1-\sigma_i)\gamma_i - \frac{1}{1-\sigma_i}((I-B')\log\lambda)_i \tag{24}$$

I call this difference as sectoral TFP measurement bias. For instance if the difference is positive, it implies an upward bias. Based on equation (24), I have the following proposition:

Proposition 4 (Measurement Bias of Sectoral TFP): *Suppose* $P_i = P_j = P_{Yi}$ *and* $\lambda_i = \lambda_j$, *we have*

$$\frac{A_{DAi}}{A_{IOi}} = \frac{\eta_i}{\lambda_i} \tag{25}$$

$$\sum_i^n \frac{A_{DAi}}{A_{IOi}} = n \tag{26}$$

On top of that suppose $\sigma_i = \sigma_j$*, then*

$$\frac{A_{DAi}}{A_{IOi}} = \frac{\mu_{Si}}{\mu_{Di}} \tag{27}$$

where $\mu_{Si} = \sum_j L_{ij}$ *and* $\mu_{Dj} = \sum_i L_{ij}$ *are intermediate input supply and demand multiplier, given* $L = (I - B)^{-1}$ *is Leontief Inverse.*

Suppose there is no gap between the sectoral gross output price deflators; no gap between the sectoral gross output price deflators and value added price deflators, and if the consumption shares are equal between sectors, the ratio of accounting method implied sectoral TFP to the model implied sectoral TFP equals the ratio of the corresponding value added share to consumption share. The sum of these sectoral ratio equals n. Suppose we also have equivalent consumption shares between sectors, the ratio be further simplified to the ratio of the corresponding sectoral intermediate input supply multiplier to demand multiplier.

Proposition 4 has important implication on the sectoral TFP measurement difference between the model implied result and the development accounting method implied result. The first prediction suggests that as long as the assumptions hold the difference mainly come from different sectoral weight. Notice that I did not impose any sectoral weight restriction when I estimate the sectoral TFP through the accounting method. But the model implied sectoral TFP measurement has strong association with the sectoral weight. Because one of the crucial implication from the observational equivalence between the input-output economy and the value added economy is that the sectoral weight in value added economy has to endogenously adjust to match the aggregate economy equivalence. The accurate sectoral value added share η_i is defined in proposition 2. The sectoral weight in the accounting method is the exogenous consumption share λ_i according to the first prediction of proposition 3. The accounting method simply assume there is no input-output linkage. We can take this case as a special form of the input-output economy when $B = 0$. If there is no input-output linkage, the sectoral weight is λ_i according to equation (11).

Even if one ignores the IO linkage, the aggregate TFP not be affected too much as long as the sectoral capital shares are close to each other. Actually if the sectoral capital shares α_i are equal across sectors, the aggregate TFP always the same. Because the aggregate capital shares in euqation (10) are equal. That is

$$\hat{\alpha}_{GO,B\neq 0} = \lambda'(I-B')^{-1}\delta_{\text{GOK}} = \lambda'(I-B')^{-1}[(1-\sigma)\circ\alpha] = \lambda'(I-B')^{-1}(1-\sigma)\alpha = \lambda'\alpha = \hat{\alpha}_{GO,B=0}$$

The third equivalence to the fourth equivalence is due to the fact that $(I-B')^{-1}(1-\sigma) = 1$. According to equation (10), equivalent aggregate capital share implies equivalent aggregate TFP. When α_i are not exactly equal but close to each other, the aggregate capital share close as well. It implies that the aggregate TFPs are close between the case with non zero input-output matrix and the case of zero input-output matrix. Since the aggregate TFPs is also equivalent between the input-output economy and the value added economy according to proposition 3, the model implied aggregate TFP is close to the development accounting method implied aggregate TFP.

Equation (26) can explain why the sectoral weight could represent the sectoral TFP measurement bias. If $A_{\text{DA}i} = A_{\text{IO}i}$ at each sector, then the summation of the ratio of these two measurement across sectors equals to n. The second conclusion implies that on aggregate level, there is little bias given the assumptions hold. However, it does not imply that each sectoral TFP would be unbiased. Given unbiased aggregate TFP, biased sectoral weight implies biased sectoral TFP measurement. Since the aggregate TFP is a sectoral weighted sum of sectoral TFP.

Suppose we further assume equivalent sectoral intermediate input share between sectors. The equivalent consumption share and intermediate input share across sectors together imply the ratio of value added share to consumption share equals the ratio of supply multiplier to demand multiplier. Therefore equation (27) suggests that, for those sectors which play a relatively more important role in supplying intermdiate inputs, the sectoral TFP measurement is likely to be upwardly biased: $A_{\text{DA}i} > A_{\text{IO}i}$. On the other hand, for those sectors which play a relatively more important role in demanding intermediate inputs, the sectoral TFP measurement is likely to be downwardly biased: $A_{\text{DA}i} < A_{\text{IO}i}$. I show whether the assumptions hold and whether the predictions exist in the next section.

3. Data

The nominal value data are available in the WIOD database. I obtain the sectoral nominal value added panel data from the SEA dataset. I obtain the sectoral nominal value of intermediate inputs from the world input-output tables (WIOT). Since I do not consider the international trade, I ignore the import and export data. Similar I also igonore the government consumption, tax/subsidy and transport margin data. For most major economies, these values are relatively small in determining the economic structure and should not affect the result systematically. But I conduct robustness check on these additional data. The sectoral nominal gross output value equals the summation of value added and the relevant intermediate input values based on equation (5) through equation (7). The capital share and intermediate input share are imputed by these nominal values based on equation (5) through equation (7) as well.

The SIRP database from Inklaar and Timmer (2014) is important to transfer the nominal values into real values. Specifically I use the time series price deflators to calculate the real value of value added, intermediate input and gross output at 2005 local price. The SEA dataset provide all the relevant value added price deflators and gross output deflators. In the model I assume no distortion between the gross output price and intermediate input price, therefore I use the gross output price delators to deflate the nominal value of intermediate input. Then I aggregtae the 35 individual sectors into 4 sectors following the table in appendix A1. Finally I use the PPP deflators from the SIRP database and the annual exchange rate from the WIOD to calculate the real gross output, real value added and real intermediate input in 2005 global reference prices.

The real primary inputs are calculated in a similar way. The SEA dataset provides sectoral real capital stock value in 1995 local price. I calculate the real capital stock in 2005 local price by using the capital price deflators in SEA. Then I use the capital PPP deflators from PWT 8.1 to calculate the real capital stock in 2005 global price. For the labour input, I use the total employment hours. Alternative choice could be number of employees. Many papers including Fadinger, Ghiglino and Teteryatnikova (2016) argues for the choice of total employment hours. Because data on hours worked allows accounting for differences in working patterns, for instance, full-time and part-time workers. I use the data on number of employees as robustness check.

The final panel data are available for 4 sectors and 39 countries over 1995-2007. The real capital data are not available in the year 2008 and 2009 for a few European countries in the WIOD 2013 database. In addition, since the SIRP database does not provide PPP price deflator data for Taiwan, I drop out Taiwan from the sample.

4. Quantitative Analysis

4.1 Quantitative Evidence of Sectoral TFP Measurement Bias

In this section I show the implication of input-output linkage on the measurement bias of sectoral value added based TFP. First I show how to empirically compare the sectoral relative TFP implied by the model and the sectoral relative TFP calculated by econometrician. Then I show the significant measurement bias in all of the four sectors. The sectoral TFP measurement covers 39 countries during 1995-2007. After that, I show where the measurement bias come from and which factor affects the measurement bias.

For the value added based sectoral TFP implied from the model, I calculate the gross output based sectoral TFP first and then transfer it into the value added based sectoral TFP according to equation (22). I compare this result with the development accounting based measurement which based on equation (12). So both methods depend on the assumption of constant return to scale Cobb-Douglas production function. In order to keep the result comparable and meaningful, I also calculate the sectoral relative TFP to the US counterpart following Fadinger, Ghiglino and Teteryatnikova (2016). It implies unitary sectoral TFP for US across the 4 sectors over 1995-2007. To save notation, I suppress the year subscript. The relative sectoral TFP is calculated by

$$\frac{A1_{ic}}{A1_{iUS}} = \frac{Q_{ic}}{Q_{iUS}} \times \frac{K_{iUS}^{(1-\sigma_{iUS})\alpha_{iUS}}}{K_{ic}^{(1-\sigma_{ic})\alpha_{ic}}} \times \frac{L_{iUS}^{(1-\sigma_{iUS})(1-\alpha_{iUS})}}{L_{ic}^{(1-\sigma_{ic})(1-\alpha_{ic})}} \times \prod_{j=1}^{n} \frac{X_{ijUS}^{\sigma_{ijUS}}}{X_{ijc}^{\sigma_{ijc}}} \tag{28}$$

Here Q_{iUS} is real gross output valued at global reference prices for sector i in US; Q_{ic} is real gross output valued at global reference prices for sector i in country c; K_{iUS} is real fixed capital stock for sector i in US; K_{ic} is real fixed capital stock for sector i in country c; L_{iUS} is total hours worked by employees for sector i in US; L_{ic} is total hours worked by employees for sector i in country c; X_{ijUS} is real intermediate input valued at global reference prices from sector j to sector i in US; X_{ijc} is real intermediate input valued at global reference prices from sector j to sector i in country c. Similar, the σ_{ijUS} and σ_{ijc} are intermediate input share from sector j to sector i for US and country c respectively. The α_{iUS} and α_{ic} are capital share of sector i for US and country c respectively. Based on this gross output based sectoral relative TFP, I can calculate the value added based sectoral relative TFP by using the equation (22).

On the other hand, econometricians may calculate the sectoral relative TFP without considering the IO linkage. Assume they calculate the relative TFP based on equation (12), which is

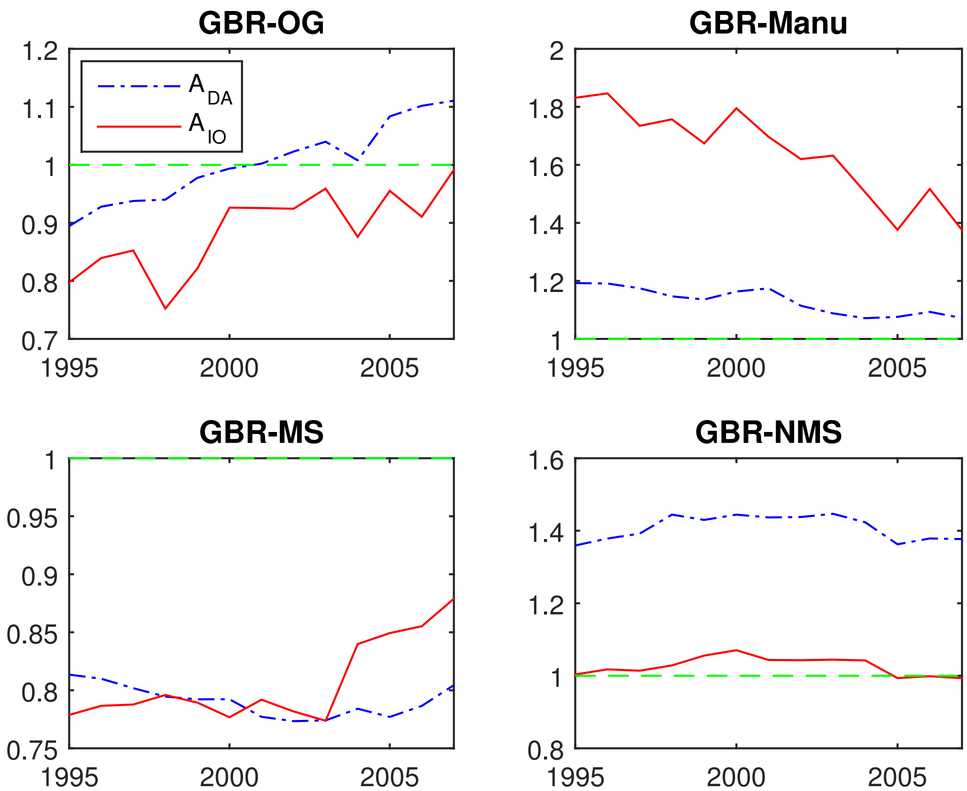
$$\frac{\widehat{A2}_{ic}}{\widehat{A2}_{iUS}} = \frac{Y_{ic}}{Y_{iUS}} \times \frac{K_{iUS}^{\alpha_{iUS}}}{K_{ic}^{\alpha_{ic}}} \times \frac{L_{iUS}^{1-\alpha_{iUS}}}{L_{ic}^{1-\alpha_{ic}}} \tag{29}$$

Here Y_{iUS} is the real value added at global reference prices for sector i in US; Y_{ic} is the real value added at global reference prices for sector i in country c. The real value of primary inputs and capital shares are calculated in exactly the same way as did in the above. However there is a measurement problem with the capital share. The capital share is calculated as a ratio of capital compensation to value added. This ratio theoretically should be nonnegative. But the data implied ratio euqals to negative value for many cases. In order to measure the TFP in a reasonable way, I assume $\alpha_{ic} = 1/3$ for all sectors and for all countries, in all years. This is a simple and common assumption in measuring TFP, such as in Jones (2015). This assumption is also generally consistent with the data. More than that, a common capital share implies the measurement is robust to different unit choice. For instance the relative TFP be the same between using 1000 as a unit or 1 million as a unit based on equation (29). Most importantly, equation (24) shows that the measurement difference between the model implied result and the development accounting result does not depend on capital share. But I consider other choices in the robustness check, such as the US sectoral capital share over time.

I present two examples of the TFP measurement result in figure 2 and figure 3. Under IO based measurement and DA based measurement respectively, figure 2 shows the value added based sectoral TFP time series result for UK. Similar figure 3 shows the corresponding result for a developing country: China.

The quantitative result suggests that there are consistent measurement bias if we use development accounting method directly. The bias could be small if we look at the market service sector in China. The bias could also be large if we take a look at manufacturing and non-market sector in UK. For most other countries, the result indicates a

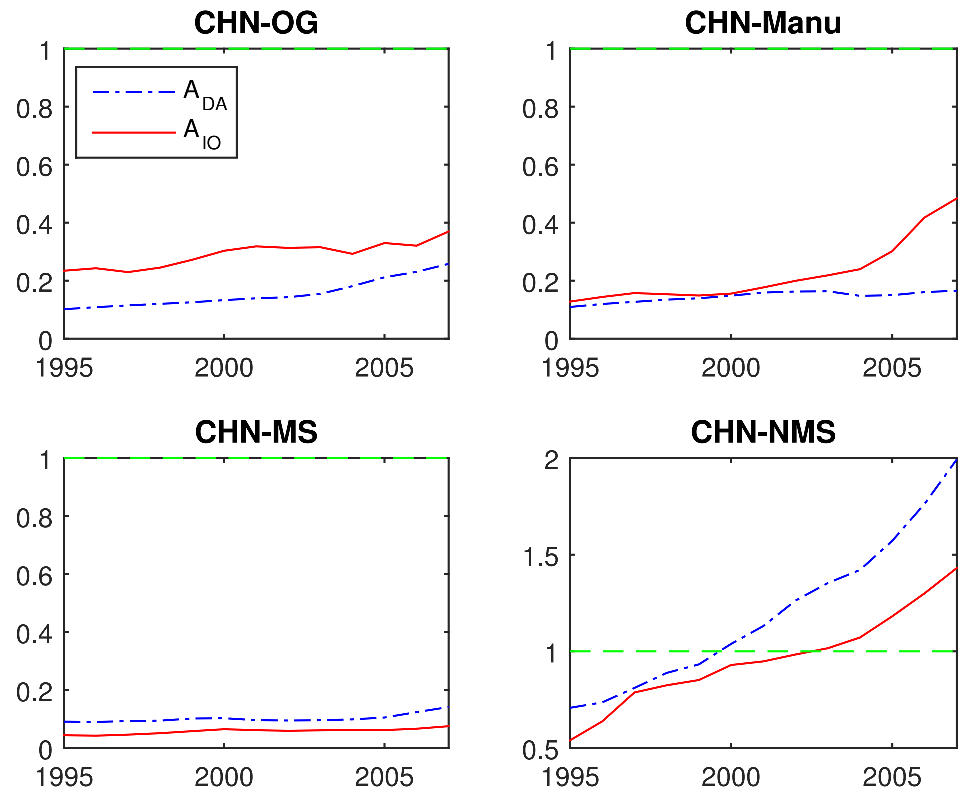
Figure 2: The relative sectoral TFP to US for UK



downward measurement bias in manufacturing sector and a upward bias in non-market service sector. Another conclusion is that the time series trend are similar in these two cases. This is not surprising because the bias should mainly come from systematically different economic structural assumption, not from the business cycle factor. I show the source of bias in the next section.

The sectoral TFP measurement result implies significant productivity difference between countries. Very surprisingly, the TFP size are remarkbly divergent between the market service sector and the non-market service sector. While the cross country market service sectoral TFPs are heterogenously distributed, the cross country non-market service sectoal TFP are relatively close. Because developed economies have remarkably larger price in non-market service sector than developing economies, which implies relatively less advantage on productivity. For example, according to Inklaar and Timmer (2014) the non-market service sectoral price of US is about 13 times higher than China.

Figure 3: The relative sectoral TFP to US for China



That is why the non-market service sectoral TFP of China possibly closes or even exceeds to that of US. Duarte and Restuccia (2016) also argues for significantly divergent cross country labour productivity comparison between market and non-market service sector. Another point to notice is that the non-market service sector is hard to measure particularly in terms of output. Follow the definition, this sector is not marketed which means very hard to price the output. For example, how to define the output of education or health care? How to price the output of household work? The measurement error could be huge in the non-market service sector.

Moerover, for developed economies like UK, their manufacturing sectoral TFP are comparable to that of US. In particular the model implied TFP result even suggest a larger manufacturing sectoral TFP of UK than US. This result happens to other developed economies like Germany, Italy, Canada and Ireland as well. For service sector and other good sector, the TFP of developed economies are lower than US for most of time.

Especially the IO based TFP is lower than the DA based TFP in market service sector, which means that developed econmies could actually have relatively lower TFP in market service sector than US. On the other hand, for developing economies the sectoral TFP are much lower than US in other good sector, manufacturing and market service sector. The cases of China can be seen in other developing economies like India, Brazil and Mexico as well. While the DA method may understate the actual TFP in manufacturing sector, it may overstate the actual TFP of market service sector for developing economies.

4.2 Source of TFP Measurement Bias

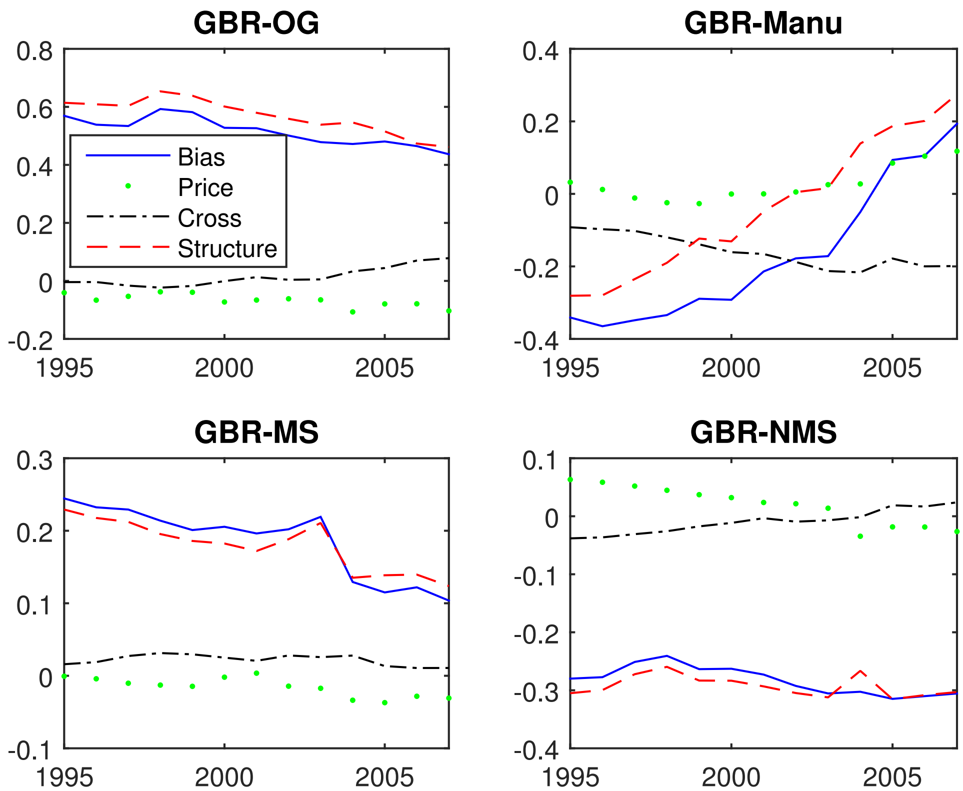
In this section I show where the sectoral TFP measurement bias come from. In order to clearly identify the source of bias, I formally decompose the difference of TFP measurement based on equation (24). Then I check which term on the right hand side of equation (24) plays a major role in affecting the measurement difference. It turns out that the input-output linkage structure mainly determines the measurement bias.

According to equation (24), the difference of sectoral TFP measurement in logarithem level come from three factors. The first term $\log P_i - \log P_{Yi}$ is the price effect which indicates the price difference between the sectoral gross output price and the sectoral value added price. Because the model implied TFP measurement uses the gross output price deflator to remove the price effect while the accounting method uses the value added price deflator to remove the price effect. Normally the difference of these two prices is small but nontrivial since the expenditure price is not exactly as same as production price. The price data from Inklaar and Timmer (2014) supports this fact.

The third term $\log(1-\sigma_i)\gamma_i - \frac{1}{1-\sigma_i}((I-B')\log\lambda)_i$ suggests that the input-output linkage affects the measurement bias. I call this term as structure term. Specifically the primary input share $(1-\sigma_i)$ and the Domar weight γ_i are determined by the input-output linkage matrix B. Equation (22) implies that IO linkage can affect the value added based sectoral TFP measurement. It suggests that if we ignore the heterogeneity of cross country input-output linkage, we may biasedly measure the sectoral TFP. The remaining second term represents the cross effect of price effect and structural effect.

For consistency I show the decomposition result of UK in figure 4; and show the decomposition result of China in figure 5. The result suggests that the bias mainly come

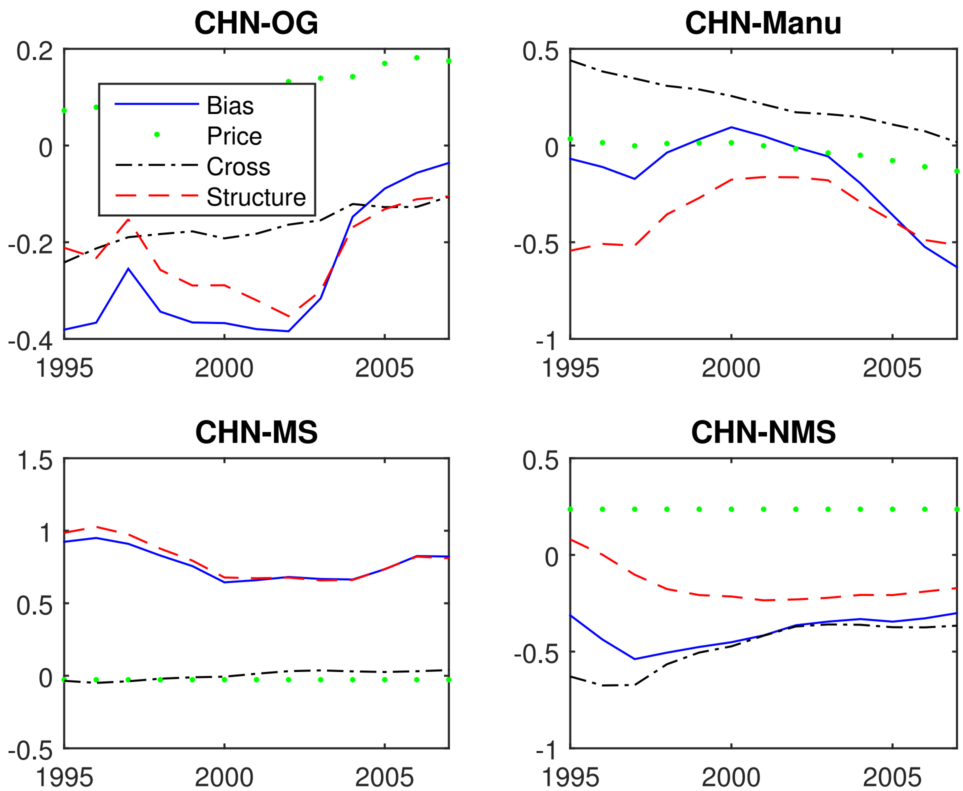
Figure 4: The Bias Decomposition for UK



from the input-output linkage structure since the time series trend of structure term (the third term) matches with the bias trend for all of the 4 sectors and for most countries. I should notice that there are exceptions in some cases. For example, figure 5 suggests that in the non-market service sector of China, the cross effect of price and structure may be more important than the structure effect in determining the bias trend. But these two examples are exceptional, we cannot see similar situation in most of other countries. The result also suggests that the price effect is small and trivial at most of time. More importantly for most cases, the price effect does not vary significantly over time.

Since the price effect and cross effect are trivial, I can ignore these two effects for analytical simplicity. It also implies that the assumption of equivalent price deflators generally holds for most sectors and for most countries. In addition, the assumption of equivalent intermediate input shares holds relatively well. For most countries, the sec-

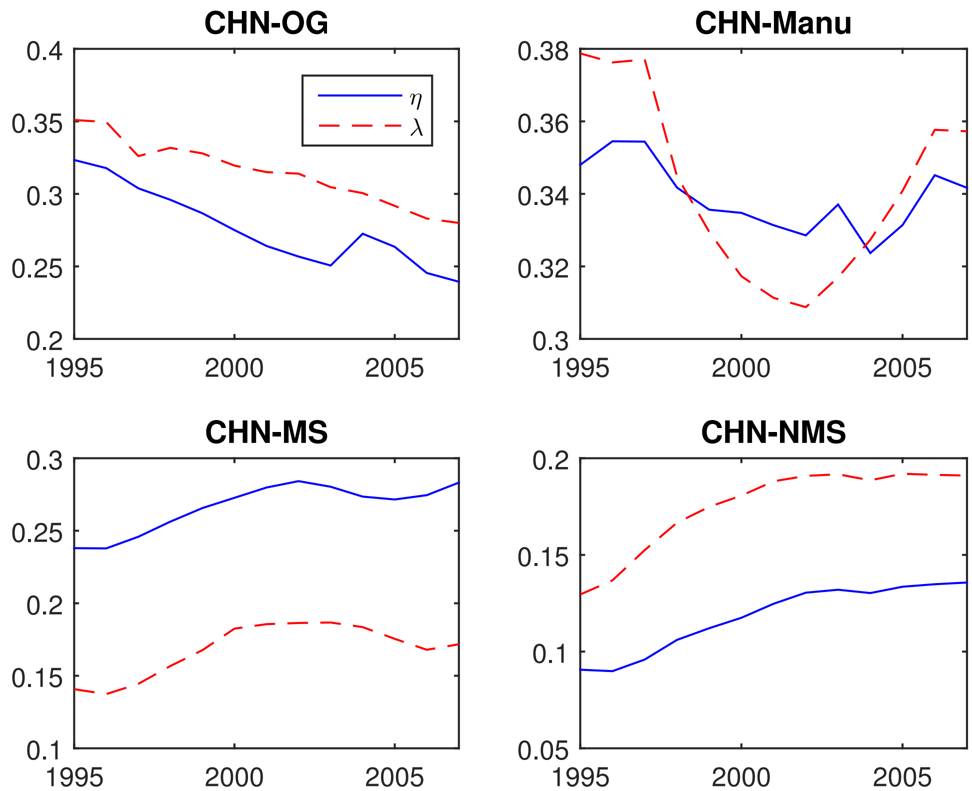
Figure 5: The Bias Decomposition for China



toral intermediate input shares σ_i are observationally close, which is consistent with the finding in Jones (2011) and Acemoglu, Ozdaglar and Tahbaz-Salehi (2017). On the other hand, the sectoral consumption shares are not close for most countries. For instance in US during 1995-2011, the consumption share of market service sector or non-market service sector is more than three times larger than the consumption share of other good sector. In effect the first and the third assumption of propostion 4 are likely to hold, the second assumption of propostion 4 hardly holds.

Though the equivalent sectoral consumption share assumption may violate, I still check whether the predictions of propostion 4 are broadly consistent with the data. The first prediction of propostion 4 suggests that any sector with higher value added share than consumption share should have overestimated sectoral TFP measurement in accounting method. I present the sectoral value added share and consumption share of China in figure 6.

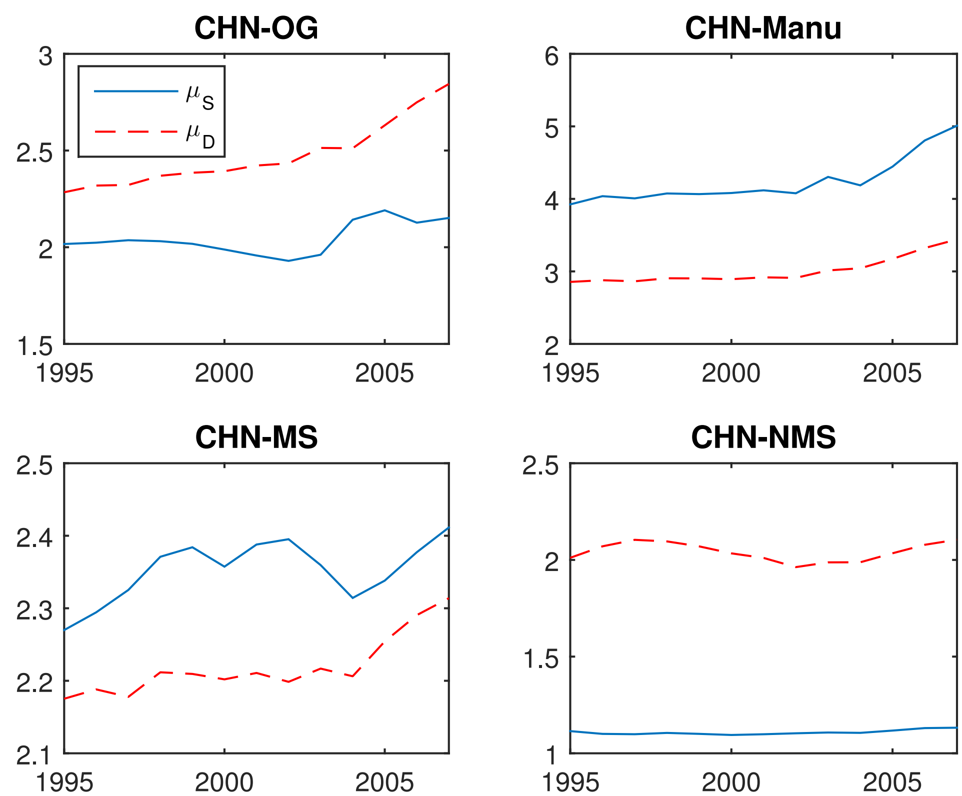
Figure 6: The Value Added Share and Consumption Share for China



The result of figure 6 tends to support the first prediction of proposition 4. For instance during 1995-2007 in the chinese manufacturing sector, the value added share is lower than the consumption share in the beginning; then it is higher than the consumption share in the middle; and lower than the consumption at the end. In the figure 5, we can observe that the bias is negative in the beginning; then it is positive in the middle; and becomes to negative again at the end. The empirical evidence broadly consistent with the first prediction of equation (25) of proposition 4 in this case.

Then I check whether the last prediction of equation (27) of proposition 4 is valid. The last prediction of proposition 4 implies that for any sector with higher intermediate input supply multiplier than demand multiplier, the DA based sectoral TFP measurement would be upwardly biased. I present the sectoral intermediate input supply and demand multiplier over 1995-2007 for China in figure 7. The empirical evidence tends to support this prediction for manufacturing sector, market service sector and non-market

Figure 7: The Intermediate Input Supply and Demand Multipliers for China



service sector. For other good sector, the prediction is not consistent with the data. The main reason of this inconsistency is that the consumption shares are not close across sectors. The first assumption is invalid for most countries. If the consumption shares are equal, the empirical result is substantially consistent with the prediction of proposition 4. The upshot is that the conclusion of proposition 4 is generally consistent with the data for most sectors and for most countries. Though there are some cases of inconsistency, due to invalid assumption of equivalent consumption shares.

4.3 Decomposition and Measurement Bias

Though the aggregate TFP are close between IO based measurement and DA based measurement, the aggregate TFP growth measurement could be different. In order to show the difference, I decompose the aggregate TFP growth. I show the decomposition bias by DA based measurement in this section. First, I decompose the aggregate

TFP growth into four components: within sectoral TFP growth; cross-sector structural change; allocation efficiency and the residual term. I show the decompostion result for the same 39 countries over 1996-2007. Second, suppose the sectoral TFPs are measured by the development accounting method. I check how does this biased measurement of sectoral TFP affect the decomposition result.

I decompose the aggregate TFP growth rate based on equation (18). The aggregate TFP in logarithm is a value added share weighted sum of sectoral value added based TFP in logarithm and the efficiency term:

$$\text{log}TFP_{VA,t} = \sum_{i}^{n} \eta_{i,t}(\text{log}A_{VAi,t} + W_{VAqi,t})$$

I use Δ to represent the time difference of a variable between year $t+1$ and year t. I have

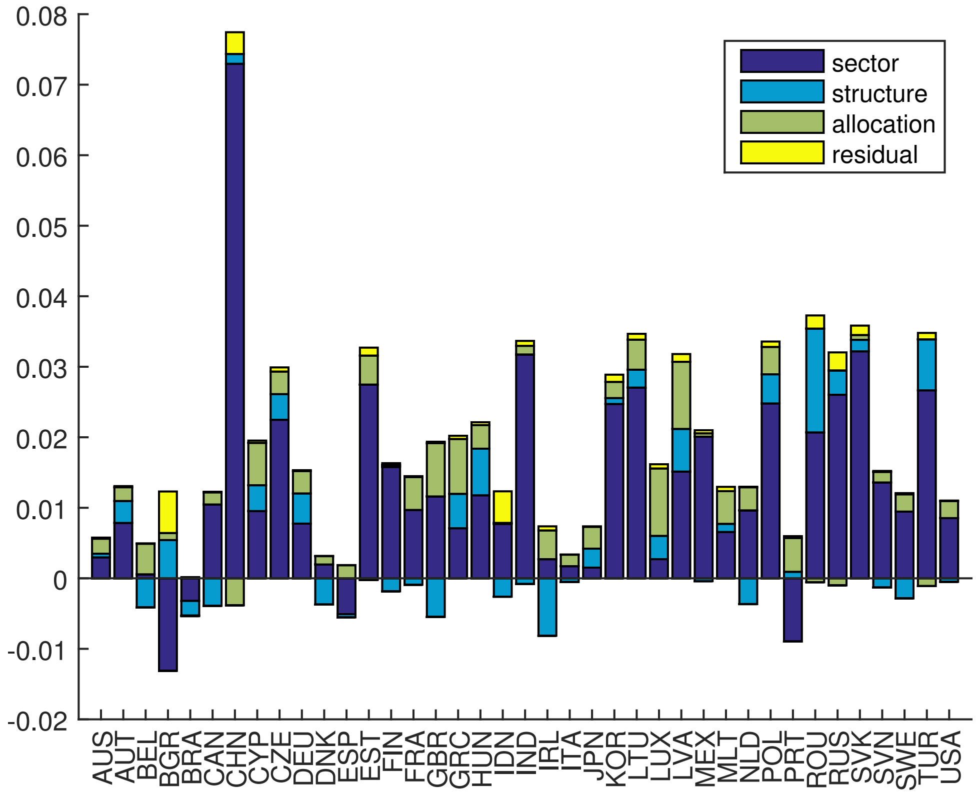
$$\Delta\text{log}TFP_{VA,t+1} = \sum_{i}^{n} \eta_{i,t}\Delta\text{log}A_{VAi,t+1} + \sum_{i}^{n} \text{log}A_{VAi,t+1}\Delta\eta_{i,t+1} + \sum_{i}^{n} \{\eta_{i,t}\Delta W_{VAqi,t+1} + W_{VAqi,t+1}\Delta\eta_{i,t+1}\}$$

When growth rate is small, the term $\Delta\text{log}TFP_{VA,t+1}$ approximately estimate the growth rate of aggregate TFP. The exact growth rate of aggregate TFP therefore equals the sum of $\Delta\text{log}TFP_{VA,t+1}$ and the residual value:

$$\frac{\Delta TFP_{VA,t+1}}{TFP_{VA,t}} = \sum_{i}^{n} \eta_{i,t}\Delta\text{log}A_{VAi,t+1} + \sum_{i}^{n} \text{log}A_{VAi,t+1}\Delta\eta_{i,t+1} + \sum_{i}^{n} \{\eta_{i,t}\Delta W_{VAqi,t+1} + W_{VAqi,t+1}\Delta\eta_{i,t+1}\} + \text{residual}_{t+1} \tag{30}$$

Equation (30) implies that the actual growth rate of aggregate TFP is decomposed by four terms. The first term is the weighted sum of sectoral TFP growth. By construction, the weight is the value added share in the last period. The second term is the sum of product of sectoral value added share change and the corresponding sectoral TFP in logarithm. This term summarises the value added share change between sectors. A negative value indicates that the economic resource are moved from relatively high TFP sector to relatively low TFP sector; otherwise the second term would be nonnegative. The third term contains the corresponding value of the first two terms, but deal with the allocation efficiency rather than the sectoral TFP. Following McMillan, Rodrik,

Figure 8: The aggregate TFP growth rate decomposition for 39 countries over 1996-2007



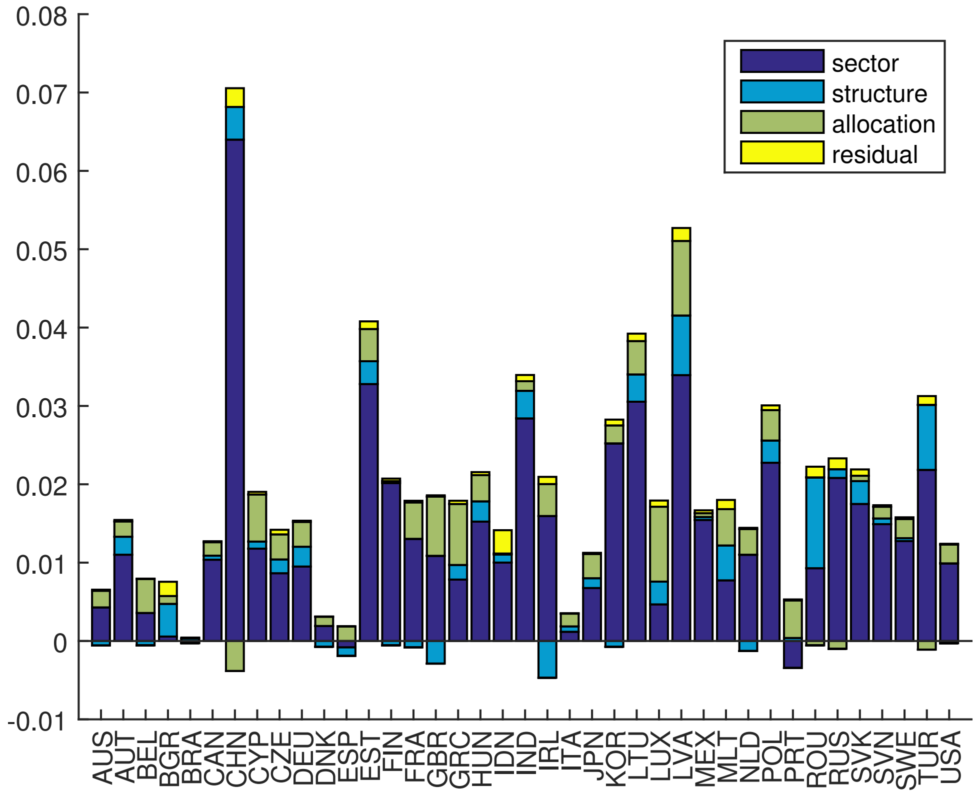
Verduzco-Gallo et al. (2014), I call the first term as within sectoral TFP growth; the second term as cross-sector structural change; the third term as allocation efficiency and the residual error term. If the annual growth rate of aggregate TFP is small, the redisual value should be small. I decompose the annual growth rate of aggregate TFP for the 39 countries over 1996-2007 based on equation (30). In order to summarise the growth decomposition result during this period, I take a geometric average on each decomposition term for all countries. This average decomposition result is shown in figure 8. The sectoral TFPs are IO based measurement A_{IO}. The horizontal axis indicates the 39 countries; and the vertical axis represents the average growth rate or contribution over 1996-2007.

Figure 8 suggests that the within sectoral TFP growth are very different between countries during this period. For most countries, the within sectoral TFP growth are positive, except for Bulgaria, Brazil and Portugal. Generally the average sectoral TFP growth rate is around 2.5 percent in developing economies; and it is around 1 percent in developed economies. But there are obvious outliers in both groups of countries. While the average within sectoral TFP growth is about 7 percent annually in China, it is less than 1 percent in Indoensia and negative in Brazil. In developed economies, the within sectoral TFP growth is larger than 1 percent in Finland and UK; but it is less than 0.2 percent in Italy and Japan; less than 0.1 percent in Belgium.

Though the size of structural change is much smaller than the corresponding within sectoral TFP growth for most countries, the cross sector structural change are heterogenous between individual countries. The most striking finding about structural change is that many countries experience significantly negative structural change. It implies that these countries move resource from relatvely high TFP sectors to relatively low TFP sectors. But their counterparts which share a lot similarities do not show negative structural change. This heterogeneity of structural change sign is robust to similar income level in developing economies if we compare China with Brazil, Indonesia and India; robust to similar income level in developed economies if we compare Germany with UK and France; robust to similar development path and cultural background (Australia vs Canada); robust to similar geography and social revolution (Hungary vs Slovenia) and also robust to similar population (Austria vs Belgium).

The allocation efficiency and the residual error term are relatively homogeneous across the 39 countries. For most countries, they are relatively small. Since there is no distortion in this study, there is little point in comparing the allocation efficiency between countries. Without distortion, the allocation efficiency only describes the value of most effective input allocation within a given country. The residual error values are very tiny for most countries, which implies the estimation and decomposition in equation (30) are accurate and reasonable. But there are a few exceptions. For instance in the case of China, the allocation efficiency is negative and the residual error is even larger than the structural change. This is due to very large variation in sectoral TFP growth, which cause the estimation relatively bad. The size of those decompostion components in these countries could be biased. But the signs should not be biased after checking at the sectoral TFP growth trend and value added share change trend. For example, the structural change in China mainly happens from other good sector to market service sector and non-market service sector. Since the average sectoral TFP in these two ser-

Figure 9: The aggregate TFP growth rate decomposition for 39 countries over 1996-2007



vice sectors is larger than the other good sector, the average structural change should be positive in China.

Now suppose the sectoral TFPs are measured by development accounting method. Given DA based sectoral TFP measurement, I decompose the aggregate TFP growth rate again according to equation (30). Since the sectoral TFPs are biasedly measured by development accounting method, the decomposition result could be biased as well. I present the development accounting implied decompostion result in 9.

Compare to the decomposition result in figure 8, the decompositin result in figure 9 are different in many places. The key differences are summarised in table 1. The first column shows the example of country code. The next two columns show the IO based within sectoral TFP growth rate measurement (unbiased) and DA based within sectoral TFP growth rate measurement (biased). Column 4 shows the difference between them. Then the following two columns show the IO based structural change measurement

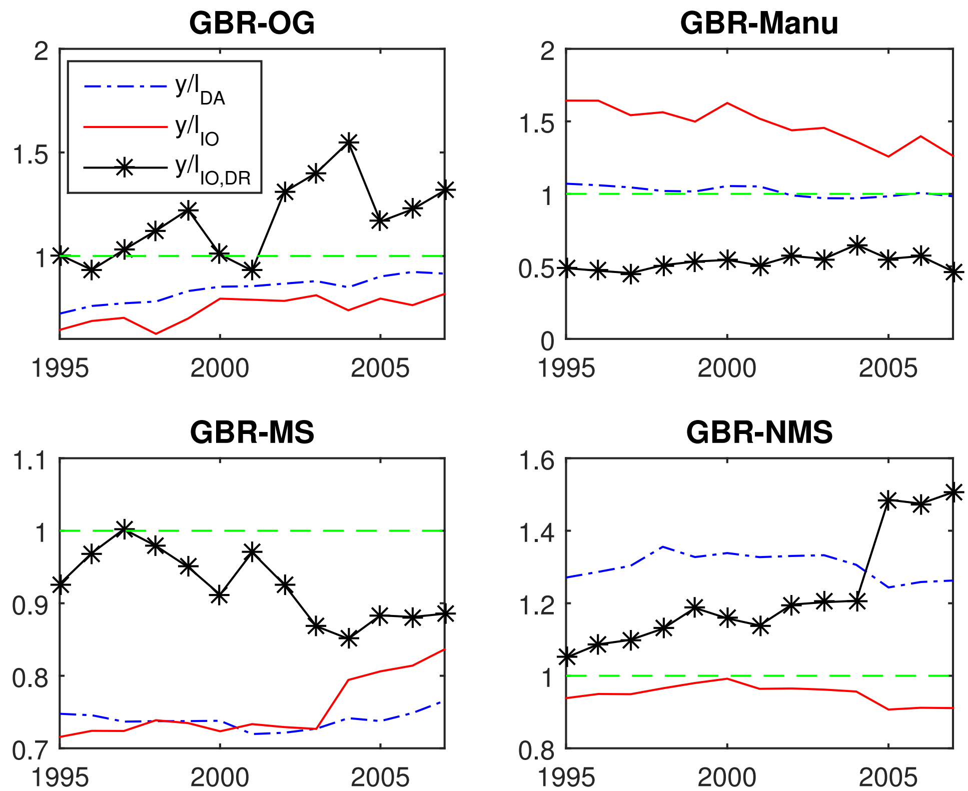
Table 1: Summary of Key Decomposition Bias (%)

(1)	(2)	(3)	(3)-(2)	(5)	(6)	(6)-(5)	(8)	(9)
Ctry Code	**Sec(U)**	**Sec(B)**	**Sec(D)**	**Stu(U)**	**Stu(B)**	**Stu(D)**	**Tot(D)**	**Tot(D%)**
LVA	1.513	3.393	1.880	0.605	0.761	0.156	2.091	65.74
IRL	0.271	1.595	1.324	-0.814	-0.468	0.346	1.703	-2202
BGR	-1.313	0.057	1.370	0.542	0.417	-0.125	0.838	-1029
SVK	3.218	1.748	-1.47	0.164	0.292	0.128	-1.394	-38.90
CZE	2.248	0.865	-1.383	0.364	0.176	-0.188	-1.571	-52.54
ROU	2.069	0.929	-1.140	1.472	1.159	-0.313	-1.503	-40.94
AUS	0.297	0.429	0.132	0.052	-0.058	-0.110	0.020	3.391
CAN	1.047	1.037	-0.010	-0.391	0.052	0.443	0.436	52.16
IDN	0.773	1.001	0.228	-0.260	0.103	0.363	0.441	45.27
KOR	2.472	2.522	0.050	0.084	-0.074	-0.158	-0.136	-4.710
USA	0.854	0.991	0.137	-0.049	-0.029	0.020	0.159	15.07
CHN	7.296	6.399	-0.897	0.139	0.417	0.278	-0.689	-9.358

(unbiased) and DA based structural change measurement (biased). Column 7 shows the difference of structural change measurement. The last column of 8 shows the measurement difference of aggregate TFP growth rate between IO based case and DA based case. Notice that the unit of all of the numbers are percentage.

There are three main differences between IO based measurement and DA based measurement. First, the structural change result switches its sign for many countries after using the development accounting method implied sectoral TFPs. This happens to 11 out of the 39 sample countries: Australia, Brazil, Canada, Estonia, Indonesia, India, Italy, South Korea, Slovenia and Sweden. It implies that the sectoral TFP measurement is important to measure the cross-sector structural change. If the sectoral TFPs are biasedly measured, we may measure the structural change incorrectly. Second, The development accounting based sectoral TFP measurement not only biases the sign of decomposition terms, but also biases the size. According to the decomposition result of figure 8 and figure 9, it biases the absolute size of within sectoral TFP growth. For

Figure 10: The relative sectoral labour productivity of UK to US



example, suppose we take the 1 percent difference as the cutoff of absolute size bias. We can find out that the development accounting implied within sectoral TFP growth of Latvia, Ireland and Bulgaria are overestimated by more than 1 percent; whereas the within sectoral TFP growth of Slovakia, Czech and Romania are underestimated by more than 1 percent. Third, given the biasness of structural change and within sectoral TFP growth, there is no surprising that the aggregate TFP growth measurement could be significantly biased by DA based sectoral TFP measurement. The last column of 1 suggest that for countries like Latvia, Ireland, Slovakia and Roumania, the aggregate TFP growth measurement are biased by more than 1 percent per annual on average.

5. Quantitative Comparison with Previous Research

In this section I quantitatively compare my study with previous study. The comparison is mainly based on counterfactual studies. There are two types of comparison. First, I compare the IO based productivity measurement in my study with IO based productivity measurement from literature. Duarte and Restuccia (2016) provides the measurement method of IO based sectoral labour productivity. I use their model implied measurement result to measure sectoral labour productivity based on the same data described in section 3. Section 5.1 shows comparison result between my IO based measurement and the replicating IO based measurement from Duarte and Restuccia (2016). Second, I check whether my IO based productivity measurement can change the result of previous literatures which measure productivity without considering IO linkage and only using DA method. McMillan, Rodrik, Verduzco-Gallo et al. (2014), Bernard and Jones (1996), Young (1995) and Herrendorf and Valentinyi (2012) are good examples of this type of literature, among other ones. I replicate their study by using my IO based productivity. Then I check the implication of this counterfactual study. Section 5.2 through 5.4 illustrate this type of comparison study.

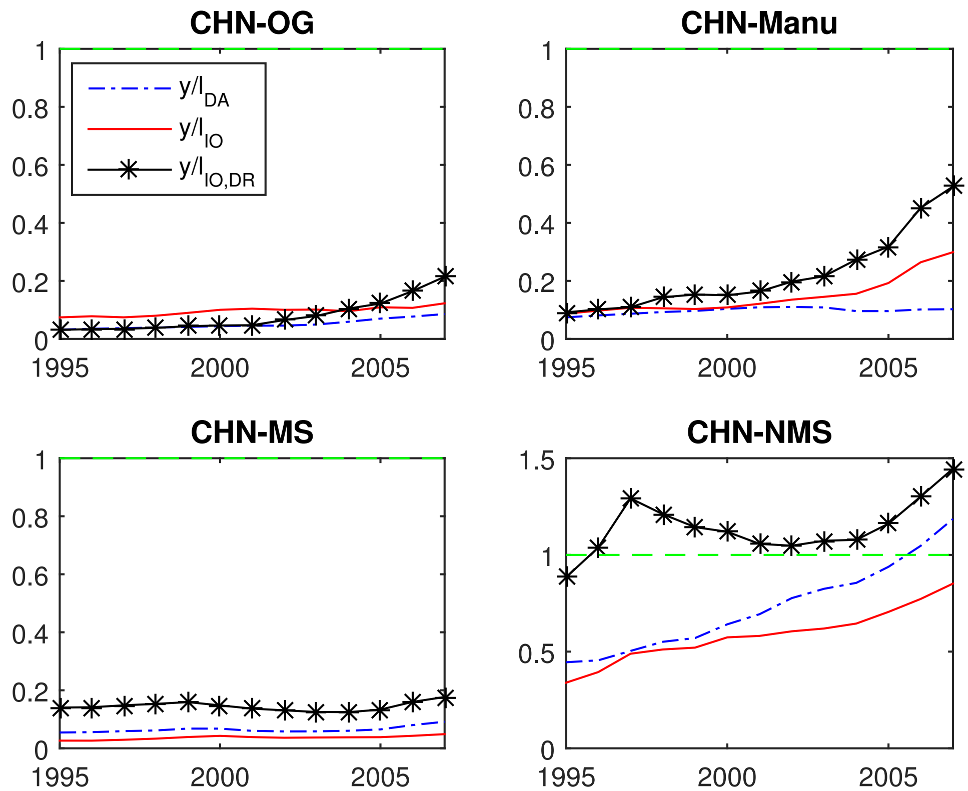
5.1 Effect of IO Linkage on Sectoral Productivity Measurement

Following Ngai and Samaniego (2009), a three sector general equilibrium model with IO linkgage is developed in Duarte and Restuccia (2016). They also consider the corresponding sectoral Value added from the gross output value equation. Then they define the relationship between VA based sectoral TFP and gross output based sectoral TFP. However, their model result actually cannot separate the value added based sectoral TFP from sectoral value added price. Because the product of these two terms equal to one large term which depends on prices and IO structure. They manually define the VA based sectoral TFP as

$$(1-\sigma_i)\log A_{\text{VA}i} = \log A_{\text{GO}i} \tag{31}$$

Compare equation (31) with equation (22), they only include part of true specification. Though Duarte and Restuccia (2016) intend to identify IO based sectoral labour productivity, they mis-specify the true IO based measurement equation form. Actually

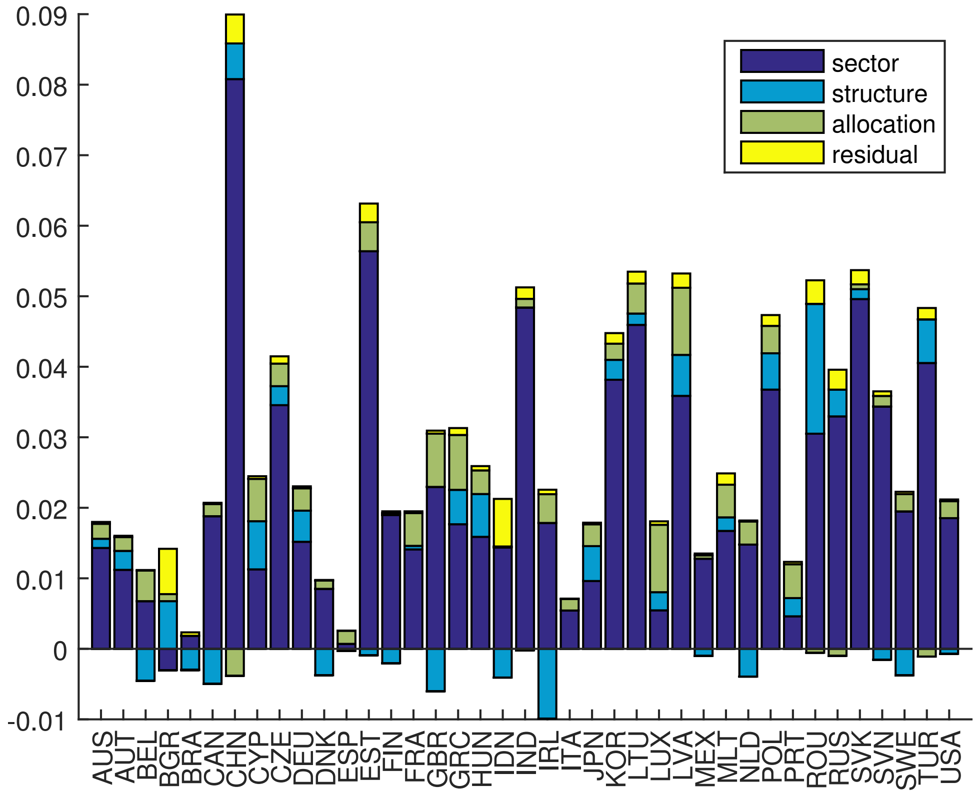
Figure 11: The relative sectoral labour productivity of China to US



Ngai and Samaniego (2009) also choose to define the value added based sectoral productivity, but their result is consistent with (22). Their function form is a special case of (22) when $\sigma_i = \sigma_j$ and $\sigma_{ij} = 0$ for any $i \neq j$.

Since Duarte and Restuccia (2016) mis-specify the true measurement function form, the sectoral labour productivity measurements under their method are different to mine. I show the examples of sectoral labour productivity measurement in figure 10 and figure 11 for UK and China respectively. The two figures show three different sectoral labour productivity measurement: DA based; IO based from equation (22) and IO based from Duarte and Restuccia (2016). All of them are different. The difference bewteen DA based measurement and my IO based measurement is consistent with section 4.1. The two figures confirm that Duarte and Restuccia (2016) have a different sectoral labour productivity measurement with my result, though both are sectoral value added based labour productivity with IO linkage.

Figure 12: Aggregate labour productivity decomposition with IO based measurement

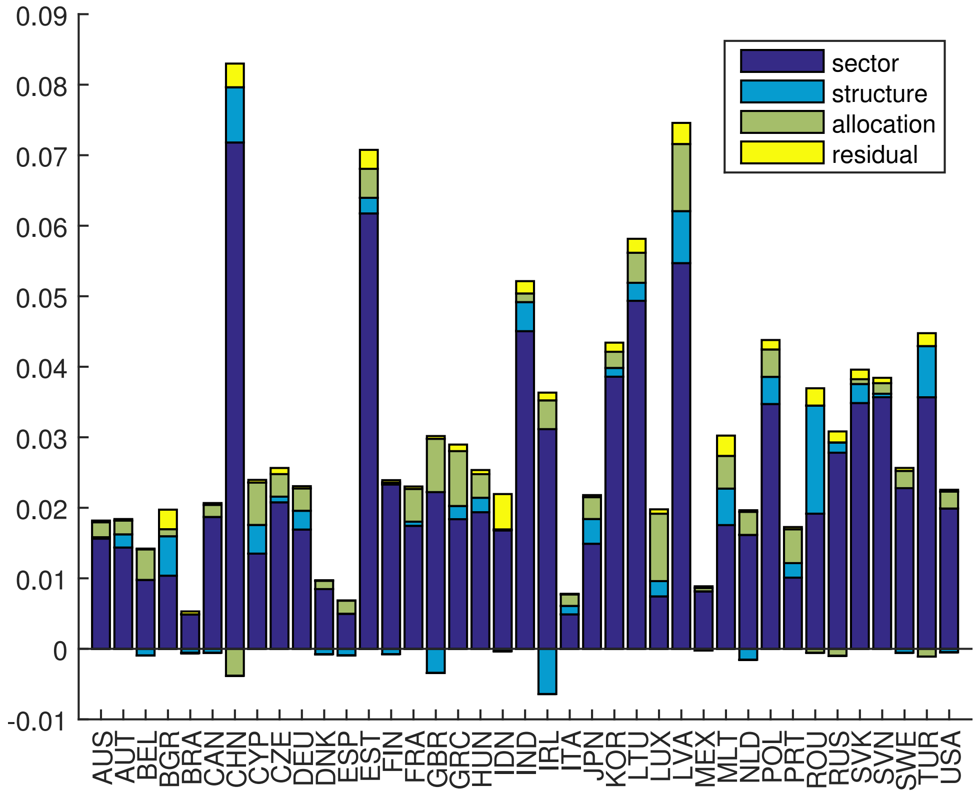


5.2 Effect of Structural Change on Aggregate Productivity Growth

McMillan, Rodrik, Verduzco-Gallo et al. (2014) argues that structural change contributes to aggregate labour productivity growth. Since countries have heterogeneous structural change, the contribution to aggregate labour productivity are heterogeneous. They decompose the aggregate labour productivity growth into within sectoral productivity growth and structural change in a similar way as equation (30). A potential probelm to their decomposition is that the labour productivity is measured by development accounting method, and without consideration of IO linkage. It implies that the sectoral labour productivity measurement could be biased, which is illustrated in the example study of UK and China in figure 10 and figure 11 respectively.

I use my IO based sectoral labour productivity measurement to decompose the aggregate labour productivity growth following equation (30). The result of this counterfatual study is presented in figure 12. I also replicate the decomposition with DA based

Figure 13: Aggregate labour productivity decomposition with DA based measurement



sectoral labour productivity measurement. The replication study of McMillan, Rodrik, Verduzco-Gallo et al. (2014) is presented in figure 13. I compare the difference of these two decomposition result.

A few example of comparison difference is summarised in table 2. Following McMillan, Rodrik, Verduzco-Gallo et al. (2014), I rank the structural change and within sectoral labour productivity growth among the 39 sample countries. The upper panel of table 2 shows the different ranking between IO based structural change measurement and DA based structural change measurement. The result is very striking. If we ignore the IO linkage, the biased sectoral productivity measurement may fundamentally cause biased understanding of the ranking of cross-country structural change. For example, according to the DA based measurement of structural change Hungary has a ranking of 17 out of the 39 sample countries, which implies a middle level of structural change. By contrast, the IO based measurement of structural change gives Hungary a ranking of 5 output of the 39 sample countries, which implies a very high level of structural change. The

Table 2: Ranking of Sectoral Growth and Structural Change

Country	**Rank**($Structure_{IO}$)	**Rank**($Structure_{DA}$)
HUN	5	17
GRC	10	18
MLT	18	6
IND	24	7
ESP	25	35

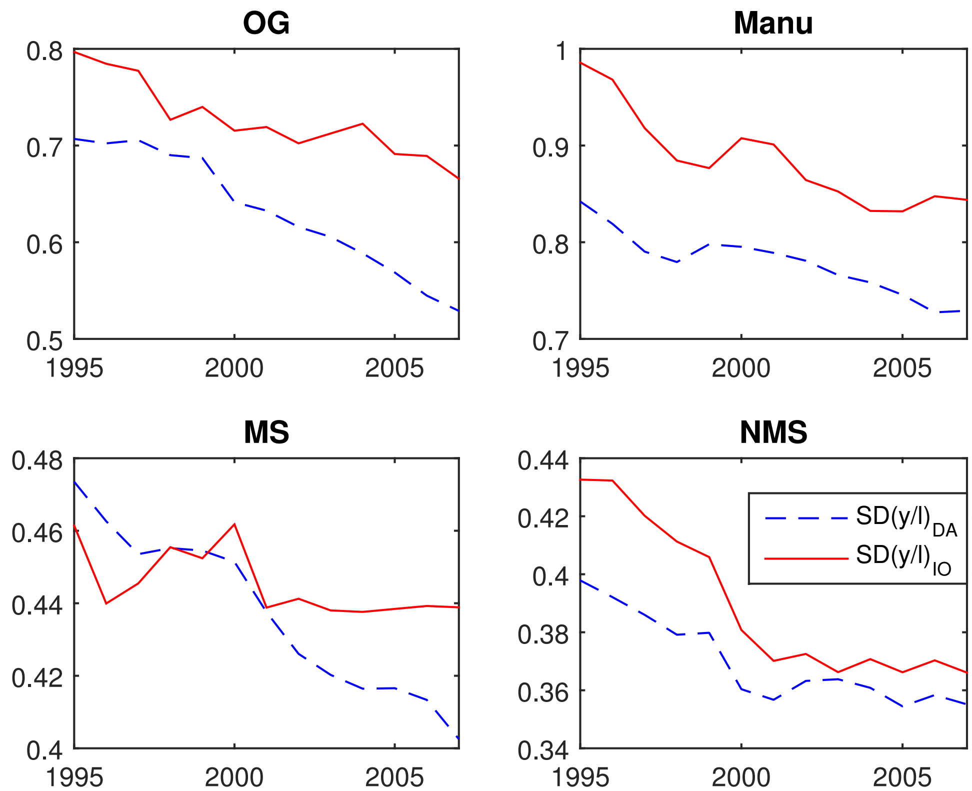
Country	**Rank**($Sector_{IO}$)	**Rank**($Sector_{DA}$)
SVK	3	9
LVA	9	3
CZE	10	16
ROU	13	19
IRL	19	11

lower panel of table 2 presents the different ranking between IO based within sectoral labour productivity growth and DA based within sectoral labour productivity growth. Similar to the structural change result, DA method also biasedly ranks the within sectoral labour productivity growth across countries.

5.3 Sectoral Productivity Convergence

Bernard and Jones (1996) studies the sectoral productivity convergence between 14 OECD countries. In their study, both labour productivity and TFP are taken into account. They want to know whether in the 14 OECD countries sectoral productivies converge to a common level over 1970-1987. In terms of convergence method, they consider two types of convergence: $\sigma-$ convergence and $\beta-$ convergence. The $\sigma-$ convergence studies whether the standard deviation of cross-country sectoral productivity shrinks over time. If the volatility of cross-country sectoral productivity decreases over time, there is evidence of convergence. On the other hand, $\beta-$ convergence studies whether the

Figure 14: The standard deviation of sectoral labour productivity across countries



initial more productive countries have less productivity growth over time. If this is true it implies that initial less productive countries grow faster than initial more productive countries, which shows evidence of catch-up for less productive countries. To be specific, they run regression of average sectoral productivity growth on initial year sectoral productivity, which is

$$\Delta \log A_i = \alpha + \beta \log A_{i,0} + \varepsilon_i \tag{32}$$

I conduct a counterfactual study to see the implication of IO based sectoral productivity measurement on sectoral convergence result in Bernard and Jones (1996). The counterfactual study is based on the 29 OECD country data over 1995-2007. The counterfactual study which use IO based sectoral productivity, compare to the replication study which following Bernard and Jones (1996) use DA based sectoral productivity. Then I want to see whether the two measurements have different convergence result.

For labour productivity, figure 14 presents $\sigma-$ convergence result for both the coun-

Table 3: Convergence Regression of sectoral labour productivity

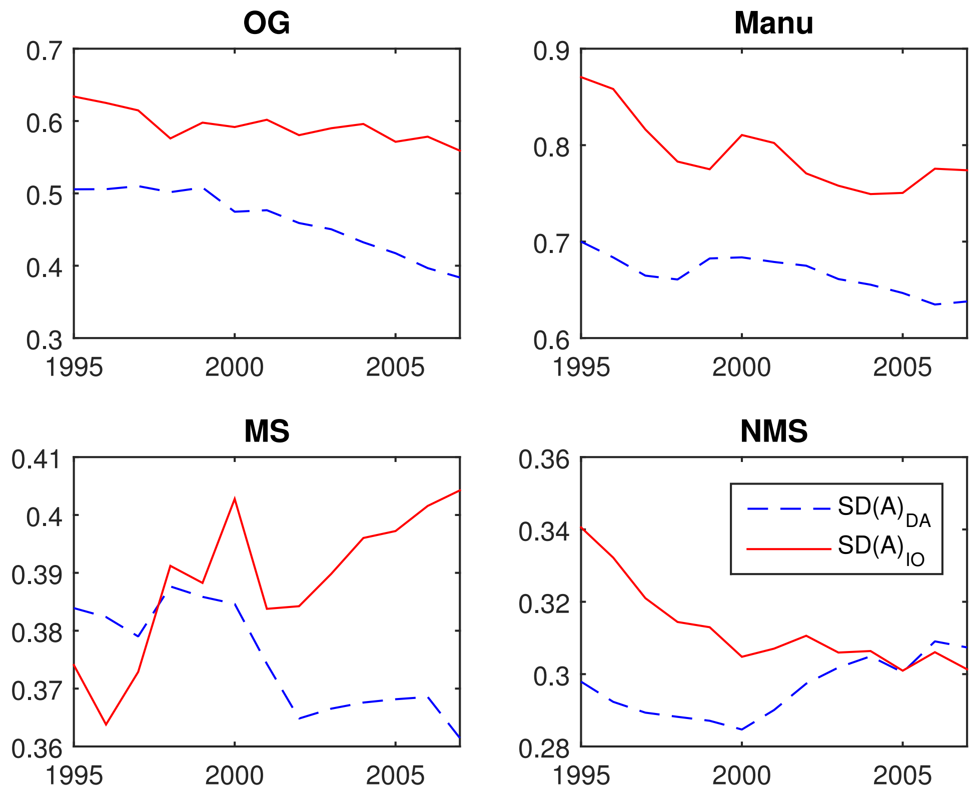
$(i)\ Dependent\ Variable : \Delta\log(Y/L)_{i,IO}$				
Sector	β	**SE**	**t**	R^2
OG	-0.013**	0.006	-2.14	0.145
Manu	-0.026***	0.009	-2.85	0.231
MS	-0.010	0.007	-1.40	0.068
NMS	-0.018***	0.005	-3.34	0.293

$(ii)\ Dependent\ Variable : \Delta\log(Y/L)_{i,DA}$				
Sector	β	**SE**	**t**	R^2
OG	-0.023***	0.003	-6.53	0.612
Manu	-0.015***	0.005	-3.20	0.275
MS	-0.018***	0.005	-3.25	0.281
NMS	-0.016**	0.006	-2.56	0.195

terfactual study (IO based) and the replication study (DA based). The most striking finding of figure 14 is that labour productivity of market service sector shows significant convergence under the DA based measurement. By contrast, IO based sectoral labour productivity measurement shows no convergence over time. Table 3 confirms this finding. The upper panel of table 3 shows the estimate of β of regression equation (32) under the IO based labour productivity measurement. The lower panel shows the corresponding case under the DA based labour productivity measurement. While the DA based estimate of β is negative and significant at 1 percent level for market service sector, the IO based estimate of β is insignificant. The result also suggests different convergence patterns for another three sectors.

For sectoral TFP, figure 15 shows $\sigma-$ convergence result for both the counterfactual study (IO based) and the replication study (DA based). While the convergence patterns are not so different between these two cases in other good sector and manufacturing sector, they are substantially different in market and non-market service sectors. In market service sector the IO based sectoral TFP measurement suggests no convergence

Figure 15: The standard deviation of sectoral TFP across countries



of TFP, whereas DA based measurement suggests evidence of convergence. The result is reversed in non-market service sector. While IO based sectoral TFP measurement suggests evidence of convergence among the 29 OECD countries over 1995-2007; the DA based measurement shows no convergence. I also present the $\beta-$ convergence result in table 4. The result confirms the comparison difference of $\sigma-$ convergence.

5.4 Cross-Country Productivity Comparison

In this section I show the implication of my IO based sectoral productivity measurement on two research studies: Young (1995) and Herrendorf and Valentinyi (2012). The former studies the aggregate and sectoral productivity performance for Four Asian Tigers economies. The latter studies which sector has larger sectoral TFP divergence. Both of them measure the sectoral productivity by development accounting method. I show whether their conclusions could be affected by using my IO based sectoral productivity

Table 4: Convergence Regression of sectoral TFP

$(i)\ Dependent\ Variable : \Delta\log(A)_{i,IO}$				
Sector	β	**SE**	**t**	R^2
OG	-0.013	0.008	-1.55	0.082
Manu	-0.026**	0.010	-2.65	0.206
MS	0.001	0.008	0.15	0.001
NMS	-0.022***	0.008	-2.79	0.224

$(ii)\ Dependent\ Variable : \Delta\log(A)_{i,DA}$				
Sector	β	**SE**	**t**	R^2
OG	-0.022***	0.004	-5.61	0.539
Manu	-0.011**	0.005	-2.29	0.163
MS	-0.010*	0.005	-1.76	0.103
NMS	-0.011	0.009	-1.24	0.054

measurement.

Table 5 presents the comparison study of Young (1995). Column 2 through column 7 shows the difference of a few average productivity variable between the DA based measurement and the IO based measurement. These productivity variables are aggregate labour productivity growth; aggregate TFP; sectoral TFP of OG, Manu, MS and NMS respectively. Apparently these differences are not trivial. For example in one of Asian Tigers economies Korea, the measurement difference of TFP growth of other good sector is about 4 percent per annual. This is a very large measurement difference. Actually the IO based average TFP growth of OG is negative 1.910 percent whereas the DA based average TFP growth of OG is positive 1.881 percent. Similar measurement differences are observable in both of aggregate productivity and sectoral productivity and in all of the example countries. The result implies that without considering IO linkage the cross country productivity comparison may misunderstood.

Follwing Herrendorf and Valentinyi (2012), I study the sectoral TFP disparity in a

Table 5: Productivity measurement difference b/w without and with IO linkage (%)

Ctry Code	D(Y/L)	D(TFP)	D(OG)	D(Mu)	D(MS)	D(NMS)
KOR	-0.135	-0.136	3.791	-0.361	0.611	-2.573
CHN	-0.696	-0.689	2.930	-4.371	-0.774	0.174
SVK	-1.411	-1.394	3.618	-8.792	2.887	1.588
CZE	-1.583	-1.571	0.581	-5.394	0.825	-1.750
ROU	-1.531	-1.503	2.613	-1.758	-3.128	-1.505
LVA	2.135	2.091	4.071	9.252	-0.103	1.256
IRL	1.724	1.703	-4.840	9.434	-3.181	2.466
BGR	0.859	0.837	-14.120	-4.227	0.013	-2.028

regression equation. Their regression equation is given by

$$\text{log}A_i^z = \alpha_i + \beta_i \text{log}(Y/L)^z + \varepsilon_i^z \tag{33}$$

Here z stands for country and i stands for either aggregate level or sector level. The term Y/L is average GDP per capita during 1995-2007; and A is average aggregate or sectoral TFP over 1995-2007. As Herrendorf and Valentinyi (2012) shows, regression equation (33) actually implies a relationship between relative TFP and relative GDP disparity to US, which is

$$\frac{A_i^z}{A_i^{US}} = \left[\frac{(Y/L)^z}{(Y/L)^{US}}\right]^{\beta_i} \frac{\text{exp}\varepsilon_i^z}{\text{exp}\varepsilon_i^{US}} \tag{34}$$

Therefore the larger estimate of β, the larger disparity of TFP relative to GDP per capita disparity.

I present the DA based and IO based regression estimate of β in table 6. Column 2 through column 6 of 6 shows the estimate for aggregate TFP and the four sectoral TFP respectively. The result suggests that there is little difference between these two cases at aggregate TFP estimate. But the sectoral disparity result could be significantly different. For example under the DA based sectoral TFP measurement, TFP of market service sector has larger disparity than aggregate TFP level. By contrast under IO based sectoral TFP measurement, TFP of market service sector has almost the same disparity

Table 6: Result for β_i

Method	agg	OG	Manu	MS	NMS
DA	0.777	0.552	1.029	0.814	0.347
IO	0.747	0.661	1.059	0.748	0.264

with aggregate TFP level.

6. Conclusion

This paper highlights the importance of input-output linkage in measuring productivity at both sector and aggregate level. By taking into account IO linkage, sectoral value added based productivity measurement is endogenous on IO linkage structure. This IO based sectoral productivity measurement derived from aggregate TFP equivalence result between IO economy and VA economy. If we ignore IO linkage and only measuring sectoral productivity by development accounting method, the sectoral productivity would be biasedly measured compare to IO based measurement. This paper also shows that this sectoral measurement bias mainly come from IO linkage structure. Given the measurement bias of sectoral productivity and sectoral productivity growth, the aggregate productivity growth could be biasedly measured as well. Because aggregate productivity growth could be decomposed by within sectoral productivity growth and cross-sectoral structural change; and the two decomposition terms are endogenous on sectoral productivity and sectoral productivity growth. I apply the model result to the cross-country and cross-sectoral data for 39 countries and 4 sectors over 1995-2007. The data confirms the model prediction. The quantitative result suggests that without considering IO linkage, the aggregate TFP growth measurement could be overestimated by about 15 percent for US and could be underestimated by 9 percent for China.

With the IO based productivity measurement on hand, I compare my result with previous research study. I find out that previous studies are weak in at least two points. First for research (Duarte and Restuccia 2016) which do takes IO linkage into account for measuring productivity, their method is insufficient to identify the true IO based productivity measurement. They point out the role of IO linkage in measuring productivity. But they fail to think comprehensively the relationship between IO economy and VA economy, which result in biased IO based productivity measurement compare to my result. Second for research (McMillan et al. 2014; Bernard and Jones 1996; Young 1995; Herrendorf and Valentinyi 2012) which do not take IO linkage into account, their productivity are biasedly measured by DA method. Therefore their result and conclusion are likely to be misunderstood.

A. Appendix: Proofs of the Propositions

This appendix contains outlines of the proofs of the propositions reported in the paper.

Proof of Proposition 1. Equilibrium Solution of IO Economy

Based on equation (2) and equation (7), I have

$$P_j C_j + \sum_{i=1}^{n} \sigma_{ij} P_i Q_i = P_j Q_j \tag{35}$$

I substitute equation (4) to equation (35):

$$\lambda_j + \sum_{i=1}^{n} \sigma_{ij} \frac{\lambda_i Q_i}{C_i} = \frac{\lambda_j Q_j}{C_j}$$

I let $v_j = \frac{\lambda_j Q_j}{C_j}$, then I have

$$\lambda + Bv = v$$

Here λ is a $n \times 1$ vector of sectoral consumption expenditure share; B is the input-output coefficient matrix $(n \times n)$ with $B'(i,j) = \sigma_{ij}$. I solve the vector v in the above equation as:

$$v = (I - B)^{-1} \lambda \equiv \gamma \tag{36}$$

where

$$\gamma_i = \frac{\lambda_i Q_i}{C_i} = \frac{P_i Q_i}{Y}$$

According to equation (36), the vector of γ is a vector of Domar weight (Domar 1961). The Domar weight is defined as a ratio of sectoral gross output value to aggregate value added, which shows the importance of a sector in the input-output economy. According to equation (35), the gross output value of a sector depend on the ability to supply final good to consumer, and most importantly the ability to supply intermediate inputs to other sectors. The larger the supply ability the larger gross value this sector deserves, and therefore the larger the Domar weight.

The equation (36) implies that the size of a sector in input-output economy depends on the corresponding row vector of the Leontief inverse matrix and the consumer's preferences. The equation (36) is rewritten as

$$\gamma_i = \sum_{j=1}^{n} l_{ij}\lambda_j$$

We can take the domar weight as the consumption share weighted average of the importance of that sector's role as an intermediate input supplier to other sectors. If the supply importances are identcial such that $l_{ij} = l_i$, the sector size equals the supply importance value $\gamma_i = l_i$. If consumer has same preferences over all sectoral consumptions, the sector size be the mean value of total supply importance (supply multiplier). Normally if the disaggregation is very high, the consumption share be substantially small which approach to the second situation. In effect, it is very likely to see a significantly positive relationship between the intermediate input supply multiplier and the domar weight.

I substitute the equation (36) to equation (7). The intermediate input is solved as

$$X_{ij} = \sigma_{ij}\frac{\gamma_i}{\gamma_j}Q_j = \theta_{ij}Q_j \tag{37}$$

Here $\theta_{ij} = \sigma_{ij}\frac{\gamma_i}{\gamma_j}$. Similar I can solve the optimal K_i and L_i by substituting the equation (36) to equation (5) and equation (6):

$$K_i = \frac{(1-\sigma_i)\alpha_i\gamma_i}{\sum_{i=1}^{n}(1-\sigma_i)\alpha_i\gamma_i}K = \theta_{GOKi}K \tag{38}$$

$$L_i = \frac{(1-\sigma_i)(1-\alpha_i)\gamma_i}{\sum_{i=1}^{n}(1-\sigma_i)(1-\alpha_i)\gamma_i}L = \theta_{GOLi}L \tag{39}$$

Here $\theta_{GOKi} = \frac{(1-\sigma_i)\alpha_i\gamma_i}{\sum_{i=1}^{n}(1-\sigma_i)\alpha_i\gamma_i}$, and $\theta_{GOLi} = \frac{(1-\sigma_i)(1-\alpha_i)\gamma_i}{\sum_{i=1}^{n}(1-\sigma_i)(1-\alpha_i)\gamma_i}$. On top of factor share, equations (37) through (39) shows that the optimal input allocation depends on the Domar weight. The sector which has higher Domar weight also have higher revenue share, therefore needs more inputs to produce.

Then I substitute the optimal intermediate inputs, capital and labour from equations (37), (38) and (39) to equation (1). The sectoral gross output is represented as

$$Q_i = A_{GOi}(\theta_{GOKi}K)^{(1-\sigma_i)\alpha_i}(\theta_{GOLi}L)^{(1-\sigma_i)(1-\alpha_i)}\prod_{j=1}^{n}(\theta_{ij}Q_j)^{\sigma_{ij}}$$

I transform the equation above in logarithm form, which gives

$$q = \text{log}A_{GO} + \delta_{GOK}\text{log}K + \delta_{GOL}\text{log}L + B'q + W_{q1}$$

Here $q_i = \text{log}Q_i$; A_{GO} is a $n \times 1$ vector of sectoral GOTFP; $\delta_{GOK} = (1-\sigma)\circ\alpha$ which is the Hadamard product of vector $1-\sigma$ and α; $\delta_{GOL} = (1-\sigma)\circ(1-\alpha)$; $W_{GOqi} = (1-\sigma_i)\alpha_i\text{log}\theta_{GOKi} + (1-\sigma_i)(1-\alpha_i)\text{log}\theta_{GOLi} + \sum_{j=1}^{n}\sigma_{ij}\text{log}\theta_{ij}$. The sectoral gross output in logarithem form q is solved as

$$q = (I-B')^{-1}(\text{log}A_{GO} + W_{GOq} + \delta_{GOK}\text{log}K + \delta_{GOL}\text{log}L) \tag{40}$$

According to the equation (36), $C_i = \frac{\lambda_i Q_i}{\gamma_i}$. The consumption vector in logarithem form is represented as

$$c = q + W_c$$

Here $c_i = log(C_i)$ and $W_c = \text{log}\lambda - \text{log}\gamma$. According to the equation (3), I solve the aggregate output in logarithem form:

$$\text{log}Y = \lambda'c = \lambda'W_c + \lambda'q = \lambda'W_c + \lambda'(I-B')^{-1}(\text{log}A_{GO} + W_{GOq} + \delta_{GOK}\text{log}K + \delta_{GOL}\text{log}L)$$

The model implied aggregate TFP with IO linkage therefore is

$$\text{log}TFP_{GO} = \lambda'W_c + \lambda'(I-B')^{-1}W_{GOq} + \lambda'(I-B')^{-1}\text{log}A_{GO} = \lambda'W_c + \gamma'W_{GOq} + \gamma'\text{log}A_{GO} \tag{41}$$

QED.

Proof of Proposition 2. Equilibrium Solution of VA Economy

According to the equation (14), $P_{Yi}Y_i = \eta_i Y$. Substitute this condition to equation

(15) and equation (16), I can solve the optimal K_i and L_i in the value added economy as

$$K_i = \frac{\beta_i \eta_i}{\sum_{i=1}^{n} \beta_i \eta_i} K = \theta_{\text{VAK}i} K \tag{42}$$

$$L_i = \frac{(1-\beta_i)\eta_i}{\sum_{i=1}^{n} (1-\beta_i)\eta_i} L = \theta_{\text{VAL}i} L \tag{43}$$

Here $\theta_{\text{VAK}i} = \frac{\beta_i \eta_i}{\sum_{i=1}^{n} \beta_i \eta_i}$; $\theta_{\text{VAL}i} = \frac{(1-\beta_i)\eta_i}{\sum_{i=1}^{n}(1-\beta_i)\eta_i}$. Then substitute equation (42) and equation (43) to equation (12), I represent the sectoral value added as

$$Y_i = A_{\text{VA}i}(\theta_{\text{VAK}i} K)^{\beta_i} (\theta_{\text{VAL}i} L)^{(1-\beta_i)}$$

I transform the sectoral value added into logarithem form, which gives

$$y = \log A_{\text{VA}} + \delta_{\text{VAK}} \log K + \delta_{\text{VAL}} \log L + W_{\text{VA}q}$$

Here $y_i = \log Y_i$; A_{VA} is a $n \times 1$ vector of sectoral value added based TFP; the term δ_{VAK} is a $n \times 1$ vector of capital share β_i; δ_{VAL} is a $n \times 1$ vector of labour share $1 - \beta_i$; $W_{\text{VA}qi} = \beta_i \log \theta_{\text{VAK}i} + (1-\beta_i) \log \theta_{\text{VAL}i}$. Based on equation (13), the aggregate value added is solved as

$$\log Y = \eta' y = \eta'(\log A_{\text{VA}} + \delta_{\text{VAK}} \log K + \delta_{\text{VAL}} \log L + W_{\text{VA}q}) \tag{44}$$

Here η is a $n \times 1$ vector of sectoral value added shares. If not consider IO linkage, the model implied aggregate TFP is

$$\log TFP_{\text{VA}} = \eta' W_{\text{VA}q} + \eta' \log A_{\text{VA}}$$

QED.

Proof of Proposition 3. Aggregate TFP Equivalence

If $\alpha_i = \beta_i$ and $P_{Yi} Y_i = (1-\sigma_i) P_i Q_i$, we have $\theta_{GOKi} = \theta_{\text{VAK}i}$. Based on the definition of θ_{GOKi} and $\theta_{\text{VAK}i}$ from equation (38) and equation (42), we have

$$\eta_i = (1-\sigma_i)\gamma_i$$

Based on equation (36), the relationship between real consumption and gross output is

$$c = q + \log\lambda - \log\gamma = q + W_c$$

According to equation (3), the IO based aggregate GDP in logarithem form is

$$\log Y = \lambda' c = \lambda' q + \lambda' W_c$$

Then based on equation (44), I have

$$\eta' \log A_{\text{VA}} = \lambda' q + \lambda' W_c - \eta'(\delta_{\text{VAK}} \log K + \delta_{\text{VAL}} \log L + W_{\text{VA}q}) \quad (45)$$

Substitute equation (45) to equation (18), the aggregate TFP_{VA} in logarithem form is

$$\log TFP_{\text{VA}} = \lambda' q + \lambda' W_c - \eta'(\delta_{\text{VAK}} \log K + \delta_{\text{VAL}} \log L) \quad (46)$$

Based on equation (40) and equation (41), the aggregate TFP_{GO} in logarithem form is

$$\log TFP_{\text{GO}} = \lambda' W_c + \lambda' q - \lambda'(I - B')^{-1}(\delta_{\text{GOK}} \log K + \delta_{\text{GOL}} \log L) \quad (47)$$

Combine equation (46) and (47), the difference of aggregate TFP under these two cases is

$$\log TFP_{\text{VA}} - \log TFP_{\text{GO}} = (\hat{\alpha}_{GO} - \hat{\alpha}_{\text{VA}})(\log K - \log L) \quad (48)$$

Here $\hat{\alpha}_{GO} = \lambda'(I - B')^{-1}\delta_{\text{GOK}}$ and $\hat{\alpha}_{\text{VA}} = \eta'\delta_{\text{VAK}}$. Given equation (19), the aggregate capital shares are equivalent since

$$\hat{\alpha}_{\text{VA}} = \eta'\delta_{\text{VAK}} = \gamma'[(1 - \sigma) \circ \alpha] = \lambda'(I - B')^{-1}\delta_{\text{GOK}} = \hat{\alpha}_{GO}$$

Therefore $\log TFP_{\text{VA}} = \log TFP_{\text{GO}}$ according to equation (48).

QED.

Proof of Proposition 4. Measurement Bias of Sectoral TFP

Given $P_i = P_j = P_{Yi}$, equation (24) is simplified to

$$\log A_{\text{AC}i} - \log A_{\text{VA}i} = \log(1 - \sigma_i)\gamma_i - \frac{1}{1 - \sigma_i}((I - B')\log\lambda)_i$$

Given $\lambda_i = \lambda_j$, we can apply the fact that

$$\frac{1}{1-\sigma} \circ ((I - B')\mathbf{1}) = \mathbf{1}$$

Substitute this fact to the equation above, we have

$$\log A_{\text{AC}i} - \log A_{\text{VA}i} = \log(1-\sigma_i)\gamma_i - \log\lambda_i$$

The definition of equation (19) implies that the ratio of TFP is

$$\frac{A_{\text{AC}i}}{A_{\text{VA}i}} = \frac{\eta_i}{\lambda_i}$$

Since the consumption share is constant, the sum of the ratio is

$$\sum_i^n \frac{A_{\text{AC}i}}{A_{\text{VA}i}} = \frac{1}{\lambda} \sum_i^n \eta_i = n$$

If we further have $\sigma_i = \sigma_j$, we can apply the fact that

$$\mu_D = (I - B)^{-1}\mathbf{1}\mu_D \circ (1-\sigma) = (I - B)^{-1}(1-\sigma) = \mathbf{1}$$

Based on this fact, the ratio of TFP is further simplified as

$$\frac{A_{\text{AC}i}}{A_{\text{VA}i}} = \frac{(1-\sigma_i)\gamma_i}{\lambda_i} = \frac{1/\mu_{Di} \times \mu_{Si} \times \lambda_i}{\lambda_i} = \frac{\mu_{Si}}{\mu_{Di}}$$

QED.

B. Appendix: Sector Classification

The sectoral classfication is presented in table 7. The 39 countries includes: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, China, Cyprus, Czech, Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Slovak Republic, Slovenia, Spain, Sweden, Turkey, United Kingdom, United States.

Table 7: Sector classification according to Inklaar and Timmer (2014)

Industry name	ISIC industry codes	Sector
Agriculture, forestry, fishing	AtB	Other goods
Mining, quarying	C	Other goods
Food, beverage, tobacca	15t16	Manufacturing
Textile products	17t18	Manufacturing
Leather, footwear	19	Manufacturing
Wood products	20	Manufacturing
Paper, printing, publishing	21t22	Manufacturing
Coke, refined petroleum	23	Manufacturing
Chemical products	24	Manufacturing
Rubber, plastics	25	Manufacturing
Non-metallic mineral products	26	Manufacturing
Basic and fabricated metal	27t28	Manufacturing
Machinery	29	Manufacturing
Electrical and optical equipment	30t33	Manufacturing
Transport equipment	34t35	Manufacturing
Other manufacturing	36t37	Manufacturing
Utiliteis	E	Other goods
Construction	F	Other goods
Motor vehicle, fuel trade	50	Market services
Wholesale trade	51	Market services
Retail trade	52	Market services
Hotels, restaurants	H	Market services
Land transport	60	Market services
Water transport	61	Market services
Air transport	62	Market services
Transport services	63	Market services
Post telecommunications	64	Market services
Financial services	J	Market services
Real estate	70	Non-market services
Business services	71t74	Market services
Government	L	Non-market services
Education	M	Non-market services
Health	N	Non-market services
Other services	O	Market services
Households with employed persons	P	Market services

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